Degrees of Freedom and the Bounds on Information Density in Chip-Less RFID

Modeling, Preliminary Results & Challenges

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Information Encoders (Visual Domain)

Information Encoders (Visual Domain)

= “A”

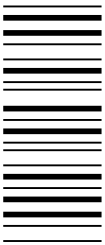
= “1 2 3”

= “EMTS”

= “HELLO”

Information Encoders (Visual Domain)

= “A”



= “1 2 3”

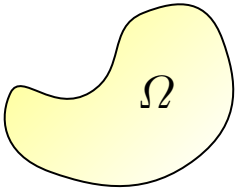
= “EMTS”

= “HELLO”

Requires direct line of sight. Detailed understanding[1].

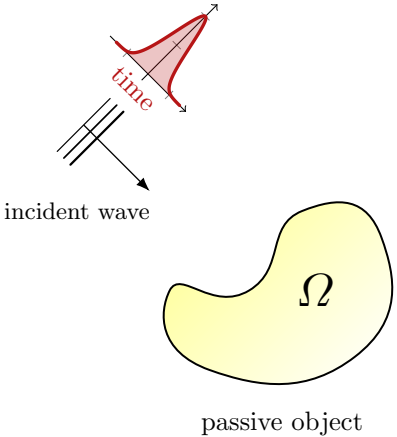
[1] G. T. di Francia, “Degrees of freedom of an image,” *J. Opt. Soc. Am.*, vol. 59, no. 7, pp. 799–804, 1969
F. Gori and G. Guattari, “Shannon number and degrees of freedom of an image,” *Optics Communications*, vol. 7, no. 2, 163–165, Feb. 1973

What About Radio Waves? (RF Domain)



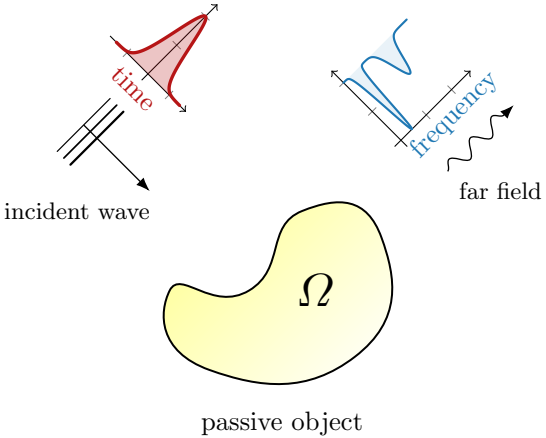
passive object

What About Radio Waves? (RF Domain)



- Solution to Maxwell's equations.

What About Radio Waves? (RF Domain)

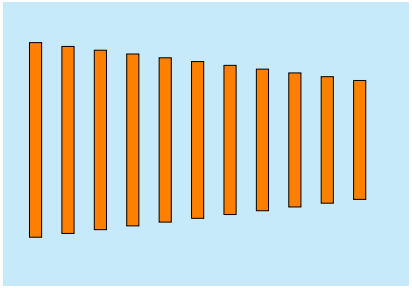


- Solution to Maxwell's equations.
- Nontrivial response.

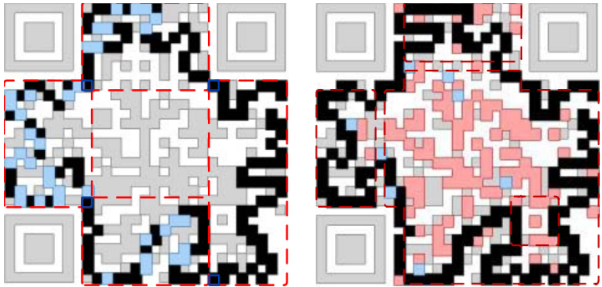
Technology: passive RFID tags[2].

[2] N. C. Karmakar, E. M. Amin, and J. K. Saha, *Chipless RFID Sensors*. Hoboken, New Jersey: Wiley, 2016
A. Vena, E. Perret, and S. Tedjini, *Chipless RFID based on RF Encoding Particle*. ISTE Press - Elsevier, 2016

Passive RFID Tags — State-Of-The-Art



Repetition & modification of a given motif[3].



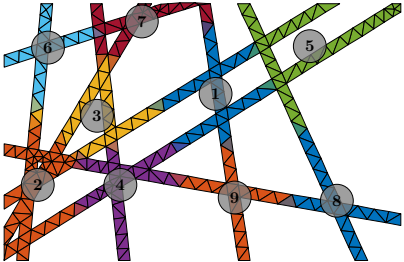
Arbitrarily-shaped resonators[4].

[3] M. Svanda, M. Polivka, J. Havlicek, *et al.*, "Platform tolerant, high encoding capacity dipole array-plate chipless RFID tags," *IEEE Access*, vol. 7, 138707–138720, 2019

[4] X. Wang, Y. Tao, J. Siden, *et al.*, "Design of high-data-density chipless RFID tag embedded in QR code," *IEEE Transactions on Antennas and Propagation*, vol. 70, no. 3, 2189–2198, Mar. 2022

Outline

1. Efficient Full-Wave Analysis
2. Bit Density
3. Preliminary Results
4. Advancements for Higher Bit Density
5. Concluding Remarks



Download the presentation from capek.elmag.org.

Question

Consider region Ω_0. How diverse can the response be across a set of scatterers $\{\Omega_i\}$ fitting it?

- Encoding capacity is function of the set of scatterers $\{\Omega_i\}$, but also of the Tx and Rx.

$$e = \frac{B}{\lambda^2 (f_2 - f_1)} \qquad B = \log_2(U)$$

- For B bits, we need at least $U = 2^B$ unique far-field responses. Exponential complexity!

C number of DOF (clusters)
S number of RCS traces to deal with, $S = 2^C$
U number of unique solutions
B number of effective bits, $B = \log_2(U)$
κ ensemble efficiency, $\kappa = U/S$

Radar Cross Section in Method of Moments

- Monostatic RCS expressed in method-of-moments' paradigm[5]

$$\rho\left(\hat{\boldsymbol{r}},\hat{\boldsymbol{e}}\right)=\lim_{r\to\infty}4\pi r^2\frac{|E_\mathrm{s}|^2}{|E_0|^2}=4\pi\frac{|\mathbf{F}(\hat{\boldsymbol{r}},\hat{\boldsymbol{e}})\mathbf{I}|^2}{|E_0|^2}.$$

[5] M. Capek, J. Lundgren, M. Gustafsson, *et al.*, "Characteristic mode decomposition using the scattering dyadic in arbitrary full-wave solvers," *IEEE Trans. Antennas Propag.*, vol. 71, no. 1, pp. 830–839, 2023

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- Current density expanded in piece-wise basis (RWG)

$$\mathbf{I}=\mathbf{Z}^{-1}\mathbf{V},\quad \boldsymbol{J}(\boldsymbol{r})\approx\sum_n I_n\boldsymbol{\psi}_n(\boldsymbol{r}).$$

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- Relation between far-field vector and plane wave excitation (monostatic RCS[5])

$$\mathbf{V}=-\mathrm{j}\frac{4\pi}{Z_0k}\mathbf{F}^{\mathrm{H}}(\hat{\boldsymbol{r}})$$

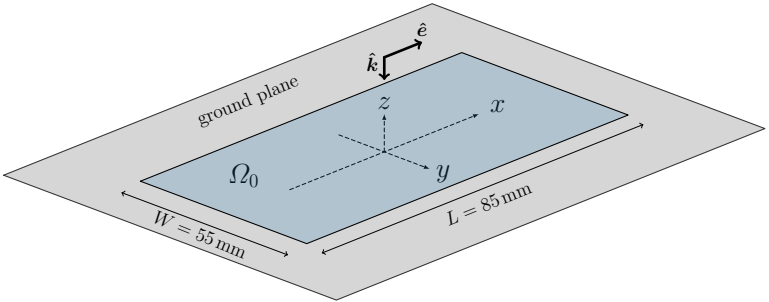
yields for $E_0=1\,\mathrm{V m^{-1}}$

$$\rho(\hat{\boldsymbol{r}},\hat{\boldsymbol{e}})=\frac{(Z_0k)^2}{4\pi}\left|\mathbf{V}^{\mathrm{H}}\mathbf{Z}^{-1}\mathbf{V}\right|^2=\frac{(Z_0k)^2}{4\pi}|\mathbf{V}^{\mathrm{H}}\mathbf{I}|^2\propto\mathcal{O}(N^3).$$

[5] M. Capek, J. Lundgren, M. Gustafsson, *et al.*, "Characteristic mode decomposition using the scattering dyadic in arbitrary full-wave solvers," *IEEE Trans. Antennas Propag.*, vol. 71, no. 1, pp. 830–839, 2023

Ground Plane Presence

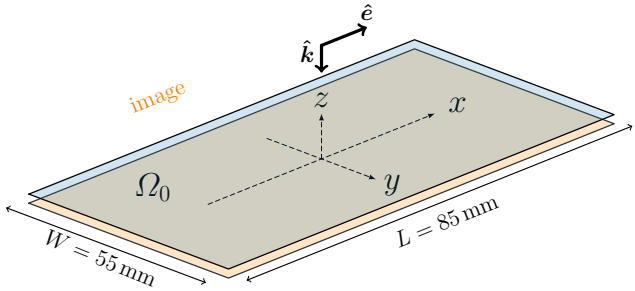
- Ground plane ⇒ $2N$ unknowns.



Ground Plane Presence

- Ground plane ⇒ $2N$ unknowns.
- Symmetry-adapted basis[6] for A'' irrep. ($E = 1$, $\sigma_\mathrm{h} = -1$) of C_s group

$$\mathbf{Z} = \mathbf{\Gamma}^\mathrm{T} \widehat{\mathbf{Z}} \mathbf{\Gamma} \in \mathbb{C}^{N \times N}.$$



[6] M. Masek, M. Capek, L. Jelinek, *et al.*, "Modal tracking based on group theory," *IEEE Trans. Antennas Propag.*, vol. 68, no. 2, pp. 927–937, 2 2020

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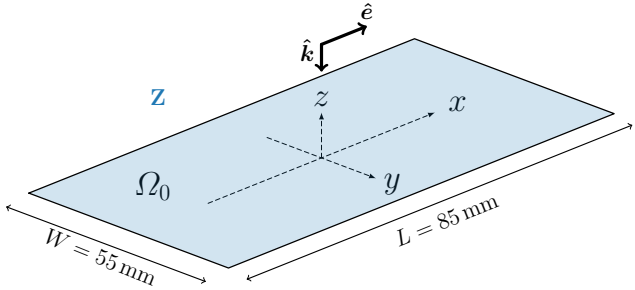
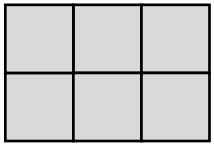


The presence of GND does not change the size of system matrix **Z**!
(The same effect as modified Green's function.)

[6] M. Masek, M. Capek, L. Jelinek, *et al.*, "Modal tracking based on group theory," *IEEE Trans. Antennas Propag.*, vol. 68, no. 2, pp. 927–937, 2 2020

Far-Field Response as a Code

Design region Ω_0



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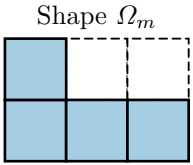
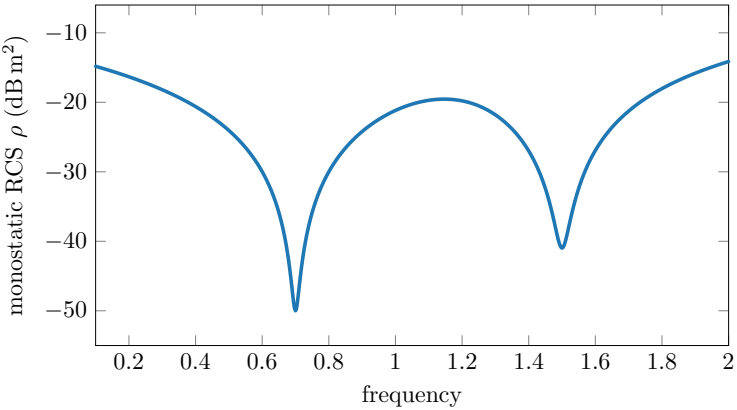
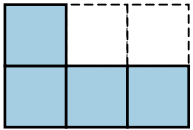
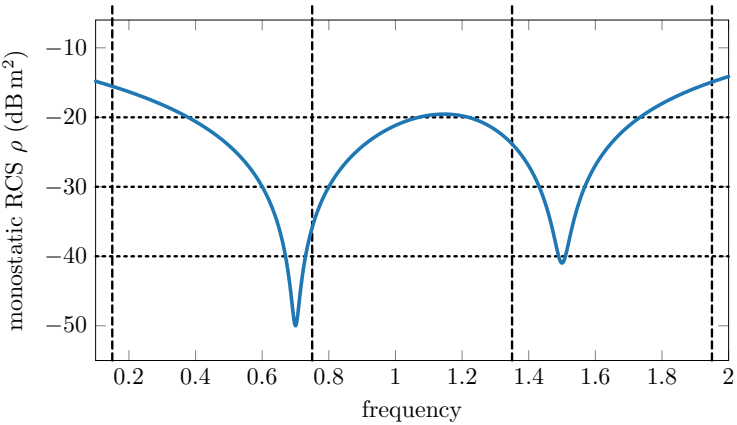




Far-Field Response as a Code

Shape Ω_m





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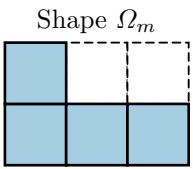

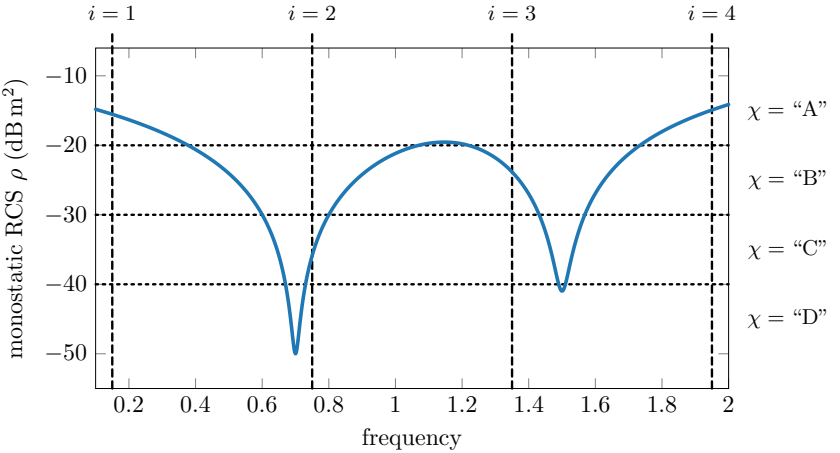

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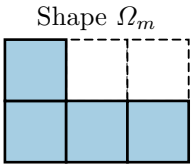

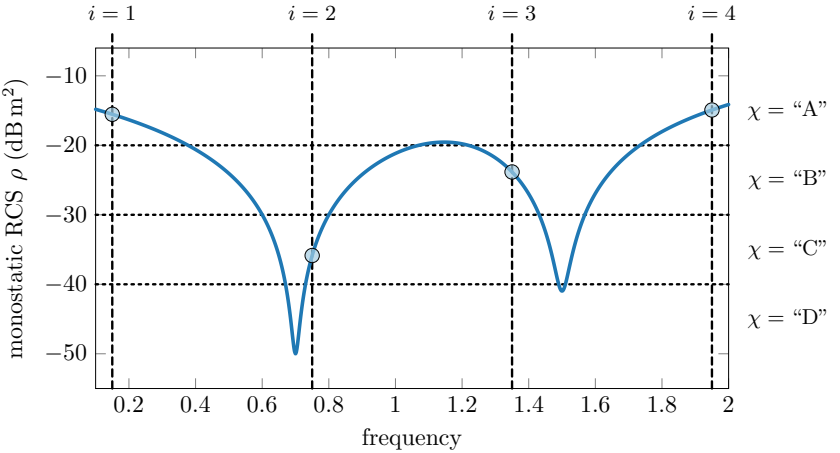

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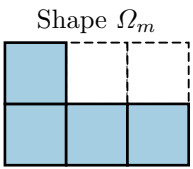

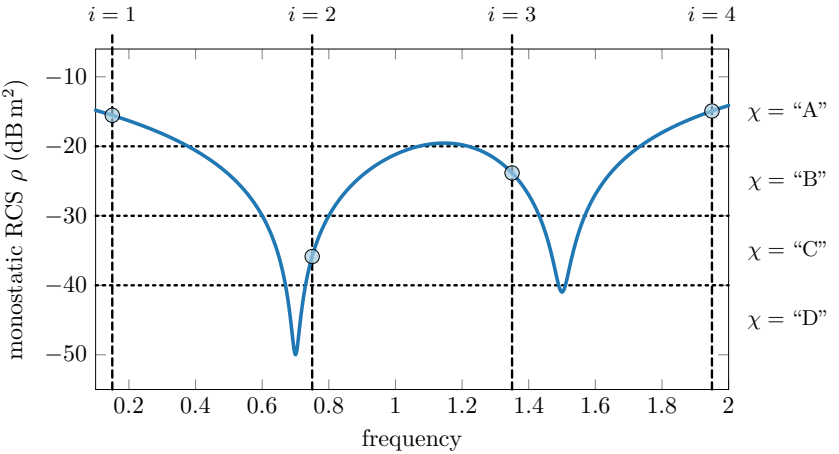

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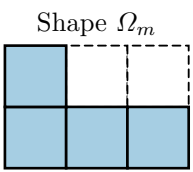

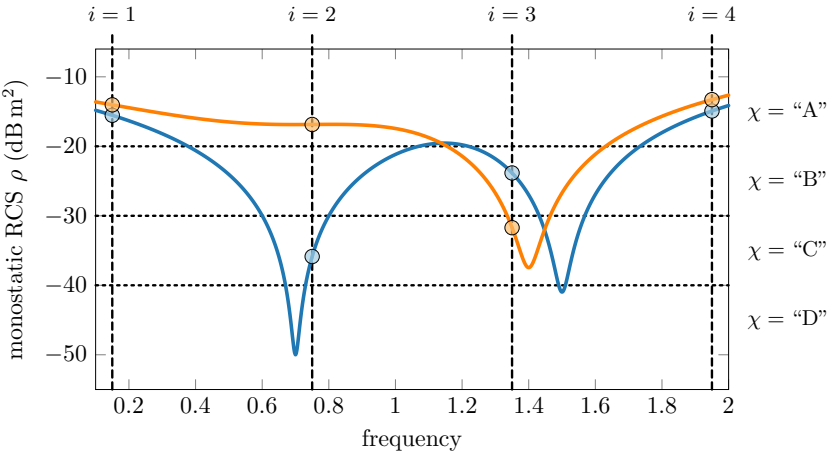

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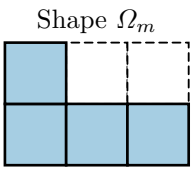
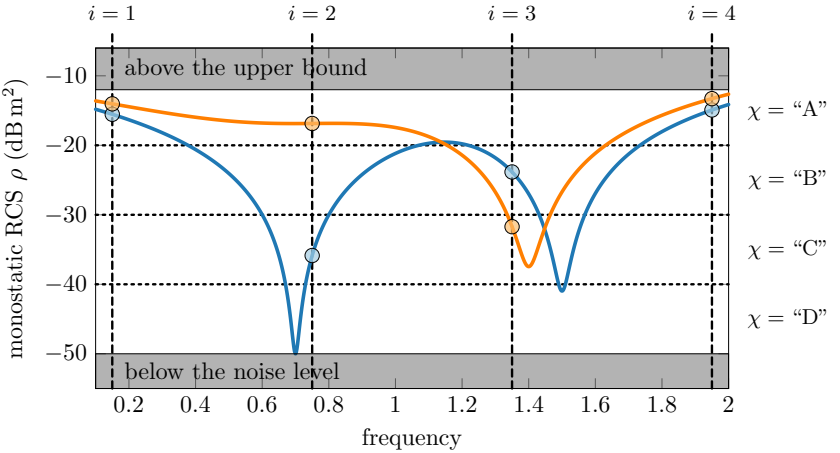


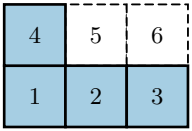


Optimization Problem

- Maximize the size of set $\mathcal{U}$ of encodes with unique RCS:

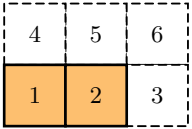
$$\begin{aligned} \text{maximize} \quad & U = |\mathcal{U}|, \\ \text{subject to} \quad & \forall m, n \in \mathcal{U},\ m \neq n \Rightarrow d(m, n) > 0. \end{aligned}$$

Shape Ω_m



χ_m = “ACBA”

Shape Ω_n



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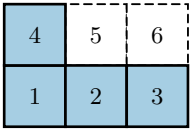
- Binary indicator characterize the encoder

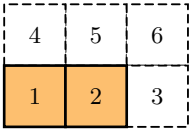
$$\mathbf{x} \in \{0,1\}^{1 \times C}.$$

$\mathbf{x}_m = 111100$



$\chi_m =$ “ACBA”

$\mathbf{x}_n = 110000$



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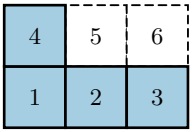
- Distance matrix

$$d(m,n) = d(\mathbf{x}_m, \mathbf{x}_n), \quad \mathbf{D} = [D_{mn}]$$

with each entry as distance $h(\cdot,\cdot)$ between symbols

$$D_{mn} = \sum_i h(\chi_i^m, \chi_i^n).$$

$\mathbf{x}_m = 111100$



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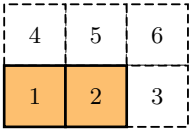

$\chi_m =$ “AACA”

$D_{mn} = 0 + 2 + 1 + 0 = 3$

Distance Matrix: Structure of Correlations

$$\mathbf{D} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \times & 1 & \mathbf{0} & \mathbf{0} & 5 & 1 \\ \times & \times & 2 & 1 & 4 & 7 \\ \times & \times & \times & \mathbf{0} & 2 & 2 \\ \times & \times & \times & \times & 1 & 3 \\ \times & \times & \times & \times & \times & 1 \\ \times & \times & \times & \times & \times & \times \end{bmatrix} \end{array}$$

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- Upper triangular matrix, rows containing zero indicate duplicity.
- Distance matrix is not needed (just take unique code words, $\mathcal{O}(N \log N)$).
- It reveals internal structure of correlations.
- Other recent works in the same direction[7].

[7] F. M. C. Nanni and G. Marrocco, "Defected laser-induced-graphene (LIG) antennas for physical unclonable function applications," in *2025 19th European Conference on Antennas and Propagation (EuCAP)*, IEEE, Mar. 2025, 1–4

Numerical Study

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- What is the best frequency sampling and quantization of RCS levels?

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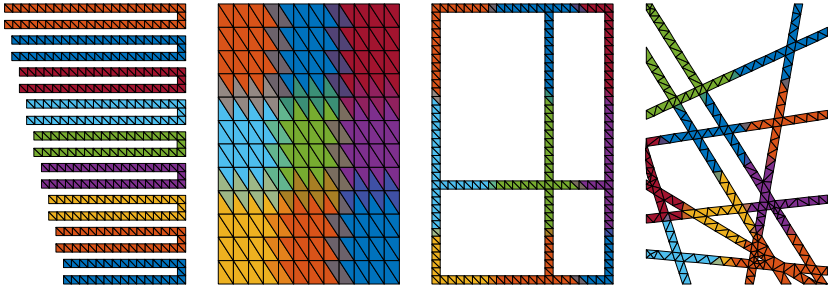
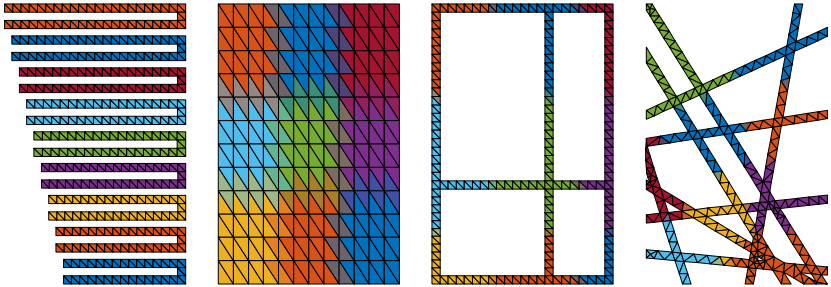


Four different layouts. Each ensemble has $2^9 = 512$ RCS responses with maximum of $\log_2 512 = 9$ bits.

Numerical Study

- What should be the layout and its clustering into shapes $\{\Omega_i\}$?
- What is the best frequency sampling and quantization of RCS levels?

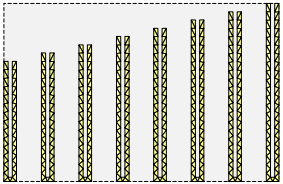


Four different layouts. Each ensemble has $2^9 = 512$ RCS responses with maximum of $\log_2 512 = 9$ bits.

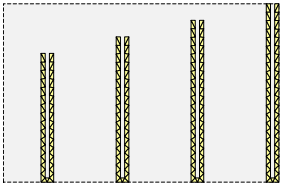
- rectangular region 85 mm×55 mm made of copper, 1 mm above GND
- $f_{\min} = 2\,\text{GHz}$, $f_{\max} = 4\,\text{GHz}$, segmentation from 1 up to 20 clusters

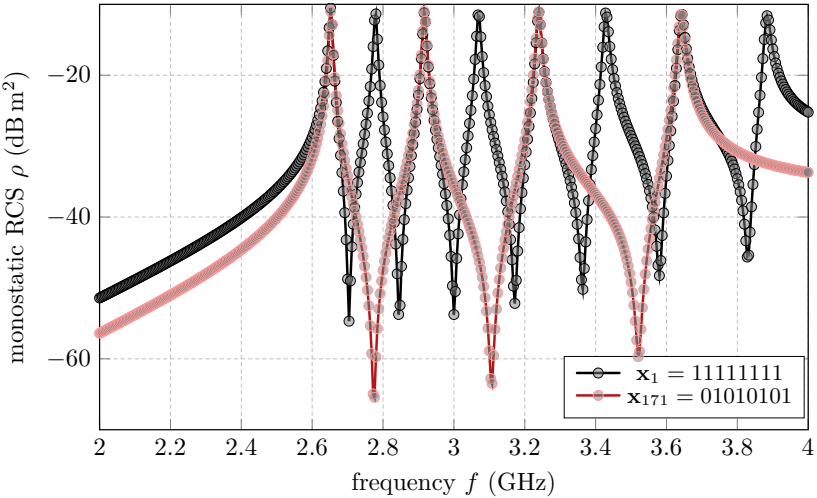
RCS Traces (Bended Dipoles)

$\mathbf{x}_1 = 11111111$

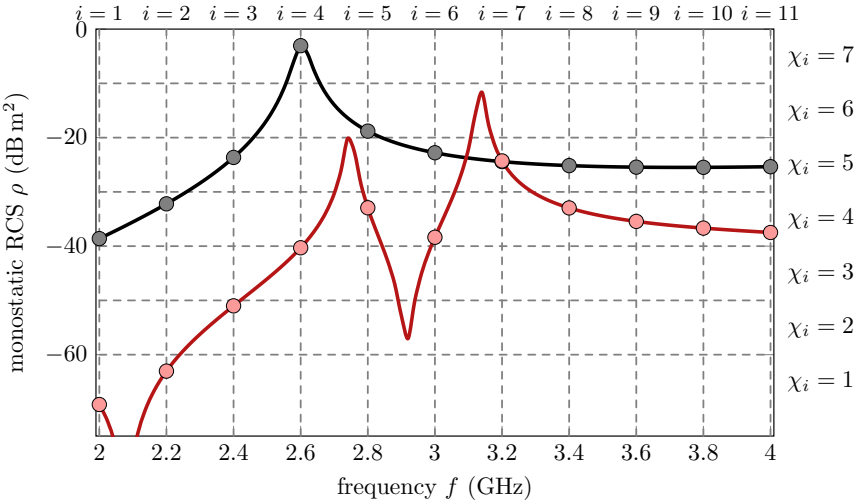


$\mathbf{x}_{171} = 01010101$





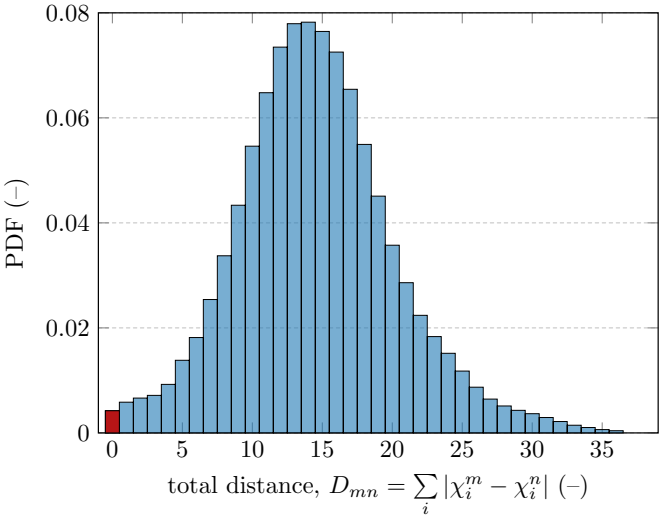
Pixel Grid (11 Frequency Samples, 6 RCS Levels)



13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

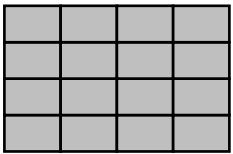
13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

Dependence on Frequency Sampling and RCS Quantization

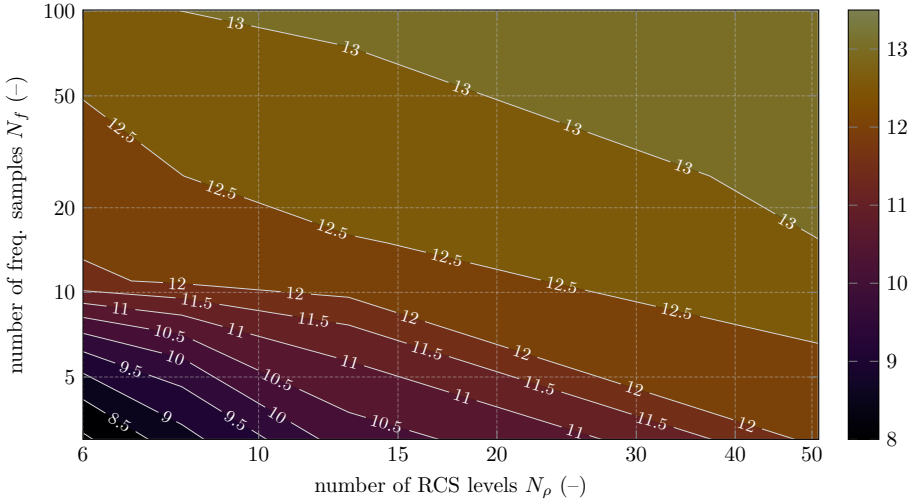


Example

- $4 \times 4 = 16$ pixels
- $S = 2^{16} = 65536$ candidates
- Symmetry-unique: 16576
- $N_f = 11$, $N_\rho = 6$ (7 symbols)
- Unique solutions $U = 3712$
- Efficiency $\kappa = 5.7\,\%$
- Effective bits: $B = 11.86$ bits

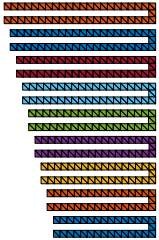
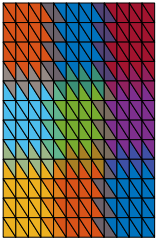
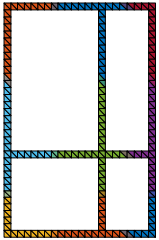
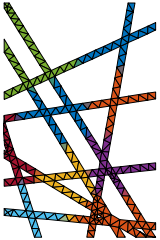


Dependence on Frequency Sampling and RCS Quantization



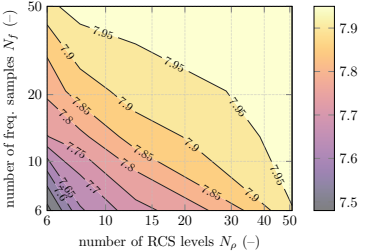
Rectangular plate (85 mm × 55 mm, 4 × 4 clusters, $f_{\min} = 2$ GHz, $f_{\max} = 4$ GHz).

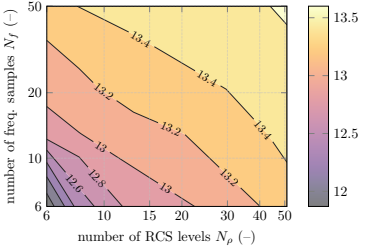
Comparison of Different Layouts (−60:10:−10 dB RCS levels)

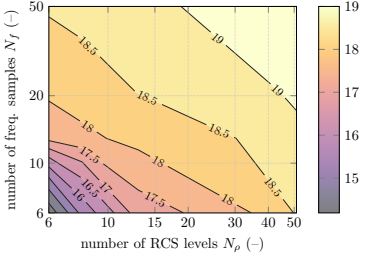





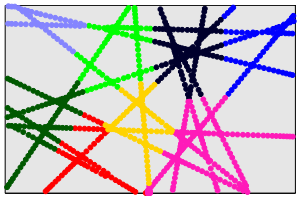
C	samples N_f	6	11	51	6	11	51	6	11	51	6	11	51
9	unique sol. κ	0.35	0.87	1	0.33	0.44	0.58	0.38	0.61	0.81	0.55	0.74	0.90
9	eff. bits B	7.5	8.8	9	7.4	7.8	7.2	7.6	8.3	8.7	8.1	8.6	8.8
16	unique sol. κ	0.01	0.15	0.99	0.03	0.06	0.09	0.04	0.24	0.42	0.06	0.26	0.43
16	eff. bits B	9.4	13	15.99	11.1	11.9	12.6	11.5	13.9	14.8	11.9	14.1	14.8

Mikado Layout — Comparison

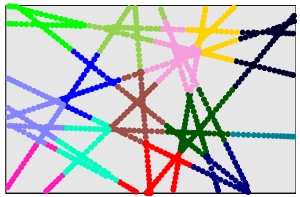




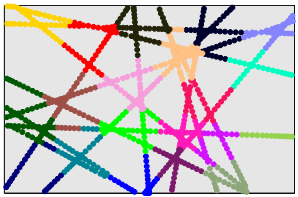




8 clusters ($C = 8$).

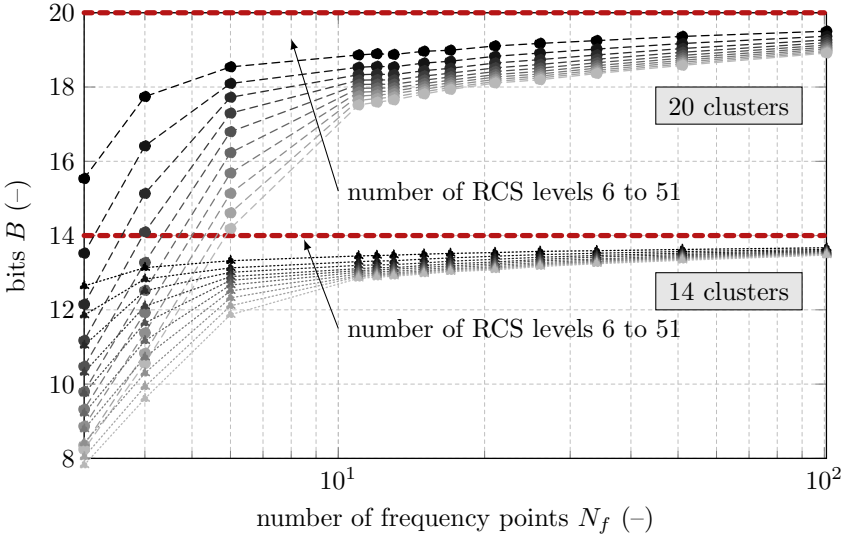


14 clusters ($C = 14$).



20 clusters ($C = 20$).

Mikado Layout — Quantization and Sampling



Large Data Sets: Monte Carlo (Mikado Layout)

- $N_f 2^C$ MoM solutions required! (*e.g.*, $C = 20, N_f = 101 \Longrightarrow 1.1 \cdot 10^8$ MoM solutions)

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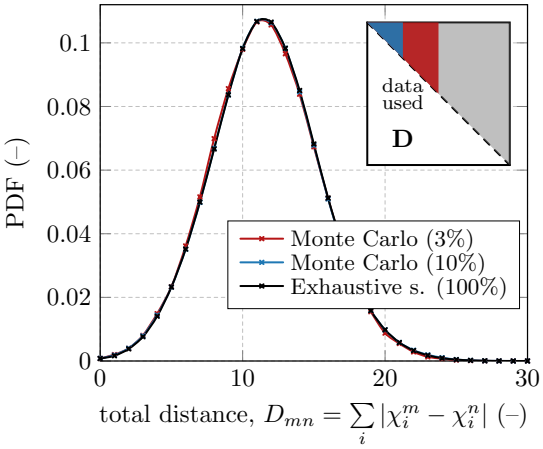

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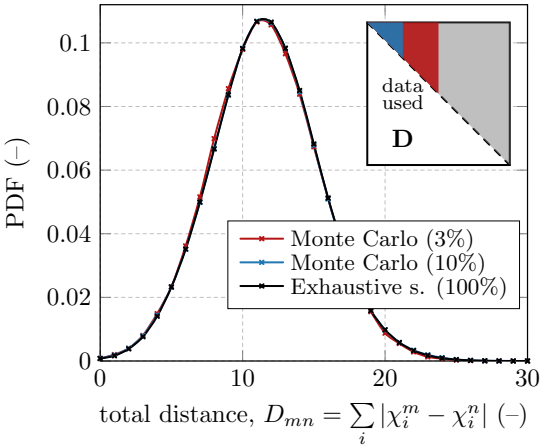
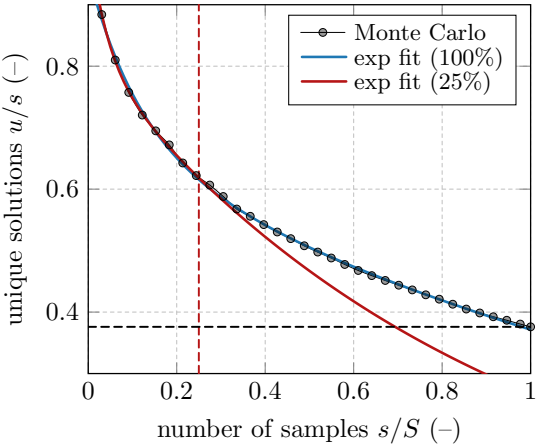
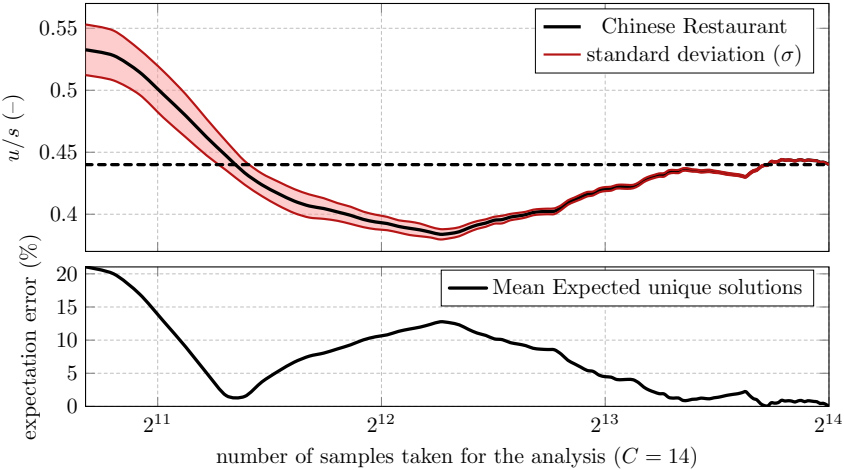




Chinese Restaurant Process

- Analogous to seating customers at tables in a restaurant (discrete stochastic process).

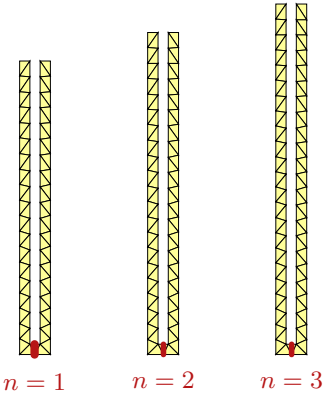


(Pseudo-)Linear-Time Evaluation

- Woodbury identity[8]

$$\mathbf{V}^{\mathrm{H}}\mathbf{Z}^{-1}(\Omega_n)\mathbf{V} = \mathbf{V}^{\mathrm{H}}\mathbf{Y}\mathbf{V} - \frac{1}{Y_{nn}}|\mathbf{y}_n^{\mathrm{H}}\mathbf{V}|^2,$$

  with admittance matrix $\mathbf{Y} = \mathbf{Z}^{-1}(\Omega_i)$.
- Designs $\mathbf{x}_i$ ordered with Gray coding (Hamming distance 1) allows for linear-time evaluation.
- Toeplitz-type MoM for uniform discretization with iterative solver and spectral preconditioner ($\mathbf{V}^{\mathrm{H}}\mathbf{I}$).



[8] W. W. Hager, "Updating the inverse of a matrix," *SIAM Review*, vol. 31, pp. 221–239, 1989
M. Capek, L. Jelinek, and M. Gustafsson, "Shape synthesis based on topology sensitivity," *IEEE Trans. Antennas Propag.*, vol. 67, no. 6, pp. 3889 –3901, 2019

Concluding Remarks

- What is the maximum information retrievable from a given region?
- Far consequences for passive RFID tag design...

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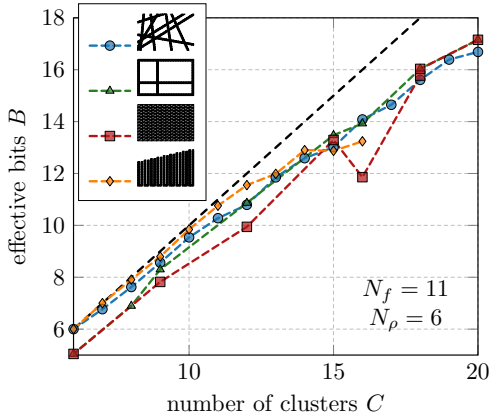
Future work:

1. statistical evaluation (CRP/...),
2. optimal layout (use of CMA?),
3. correlation between $\mathbf{x}_i$ and χ_i,
4. bounds ($> 23 \cdot 10^3$ bits for 40-L QR tags),
5. coding,
6. calibration,
7. error correction, ...



Questions?

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version 1.0
The presentation is available at ▶ capek.elmag.org

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