Inverse Design With Multi-Objective Topology Optimization Based on Binary Rank-1 Perturbations

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Stockholm, Sweden

Multi-Objective Topology Optimization in Electromagnetism[1]

- We want to optimize several parameters at once.

[1] P. Kadlec and M. Capek, *Multi-objective memetic algorithm with adaptive weights for inverse antenna design*, 2025, https://arxiv.org/abs/2409.14245.

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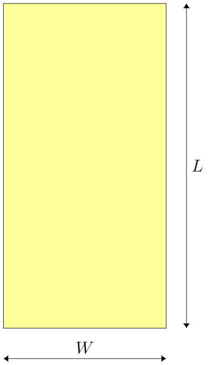

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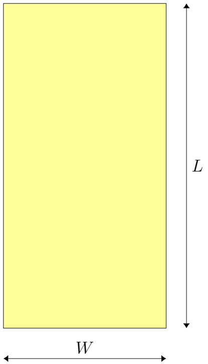

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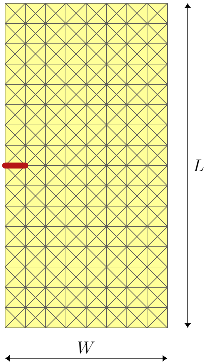

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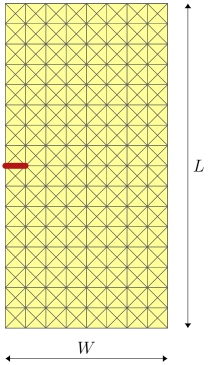

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- When to terminate the optimization?
  - Utilization of fundamental bounds.

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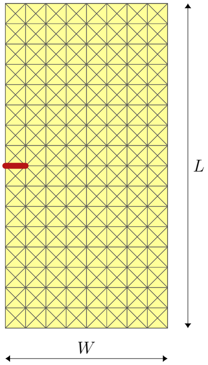

$$F^* = \{f_1^*, \cdots, f_n^*\}$$

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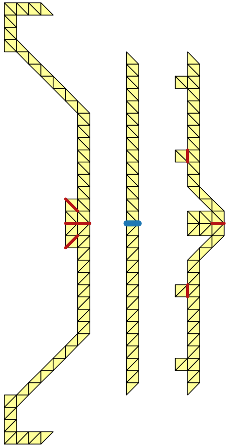
Outline

1. Novel Procedure
2. Diversity and Convergence
3. Examples
4. Concluding Remarks



Download the presentation from: capek.elmag.org.

Pareto Efficiency

Consider we want to minimize

$$\min F = \min \{f_1, f_2\} .$$

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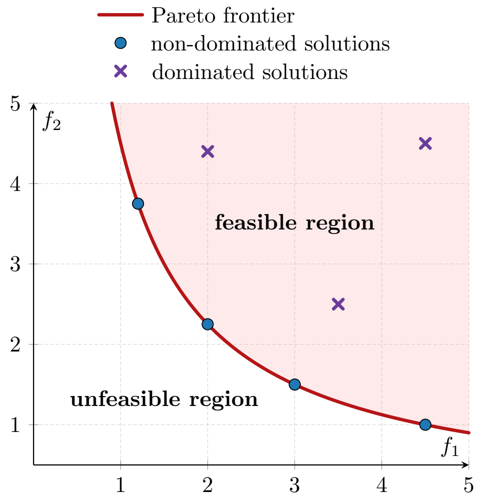
- Multiple parameters ⇒ trade-offs
- Optimal trade-off: Pareto frontier[2]



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Pareto Efficiency

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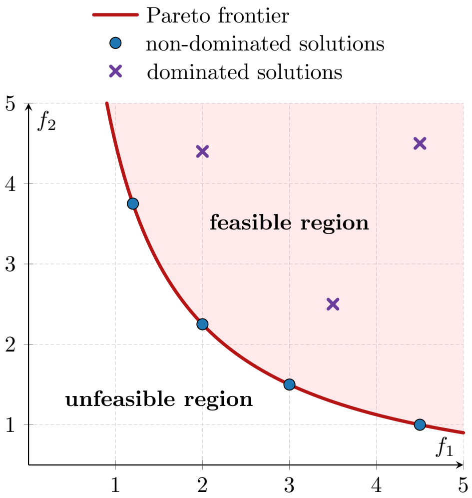
To take care of...

- convergence × diversity,
- number of non-dominated solutions,
- uniformity of Pareto frontier,
- hypervolume.



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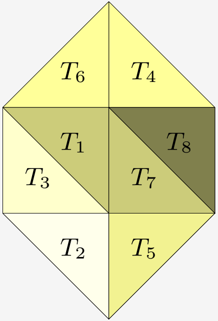
Topology Optimization: Degrees of Freedom

Density-based formulation

$$\rho_m(\boldsymbol{r} \in T_m) = [\rho_{\mathrm{min}},\ \rho_{\mathrm{max}}]$$



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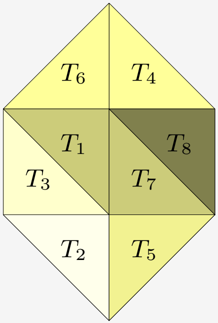
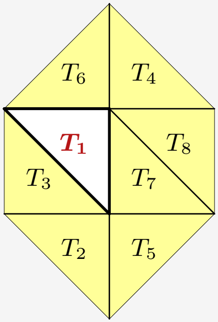


"Pixeling"

$$\rho_m(\boldsymbol{r} \in T_m) = \{0,\ \infty\}$$



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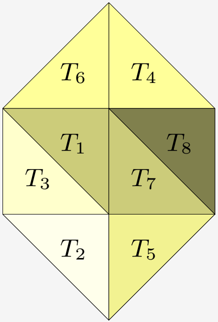

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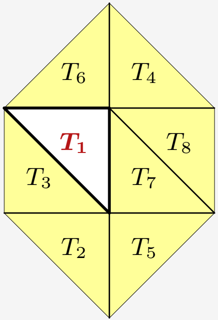
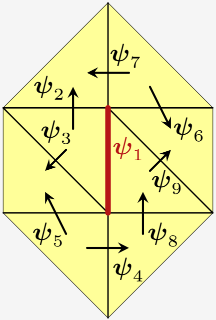


Exact reanalysis[3]

$$R_{nn,\rho} = \{0,\ \infty\}$$



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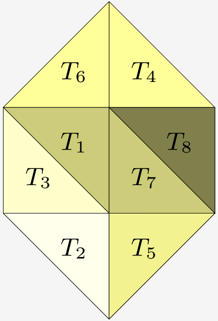

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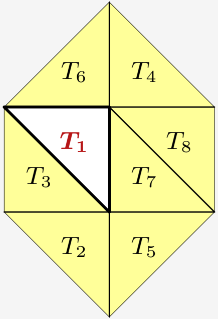

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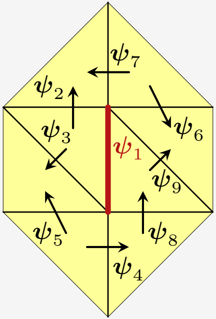

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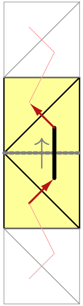
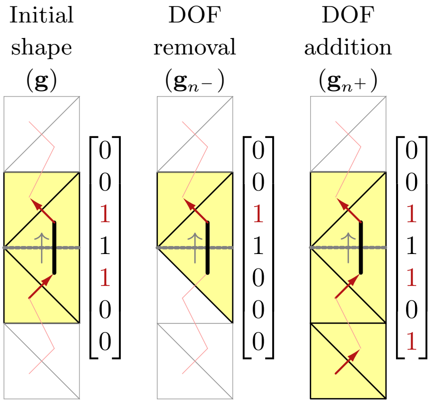
Initial shape ($\mathbf{g}$)



$$\begin{bmatrix} 0 \\ 0 \\ \color{red}{1} \\ 1 \\ \color{red}{1} \\ 0 \\ 0 \end{bmatrix}$$

Exact Reanalysis: Topology Sensitivity

- Method-of-moments impedance matrix is $\mathbf{Z}$.
- Basis function switch (removal/addition).
- All evaluations (each $\propto \mathcal{O}(N)$) at once with vectorization (Woodbury identity).



$$\begin{bmatrix}0\\0\\1\\1\\1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\\1\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\\1\\1\\0\\1\end{bmatrix}$$

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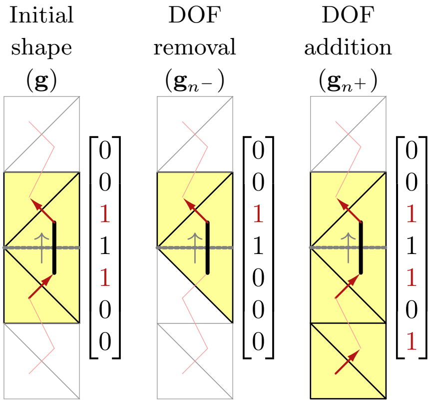
- Topology sensitivity[4]:

$$\tau_{in} = f_i\left(\mathbf{I}\left(g_n\right)\right) - f_i\left(\mathbf{I}\left(\neg g_n\right)\right)$$



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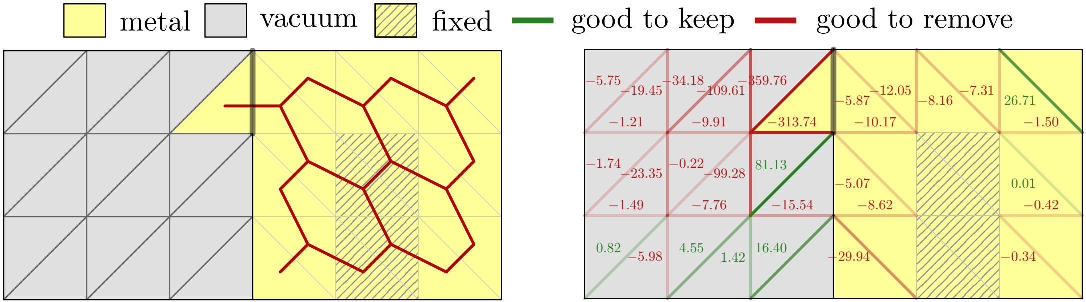
Local Greedy Algorithm Over Nearest Neighbors



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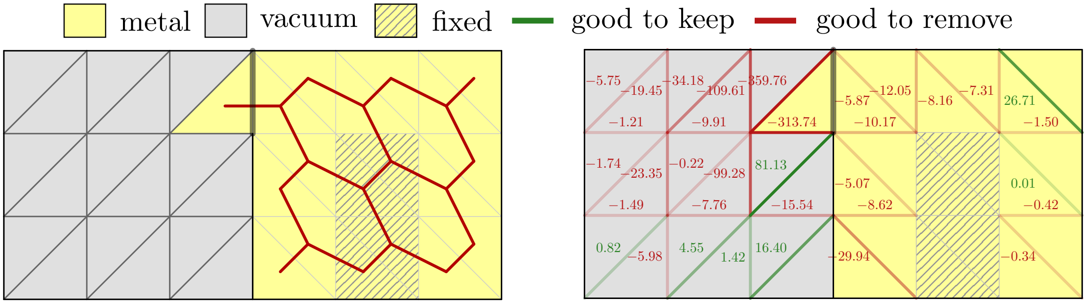

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- Local algorithm integrated into global (heuristic) scheme: powerful memetics[5].

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Dealing With Multiple Objectives — Scalarization

- What about multiple objectives?
  Common approach: scalarization:

$$f = \sum_i w_i f_i = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{f}, \quad \sum_i w_i = 1$$

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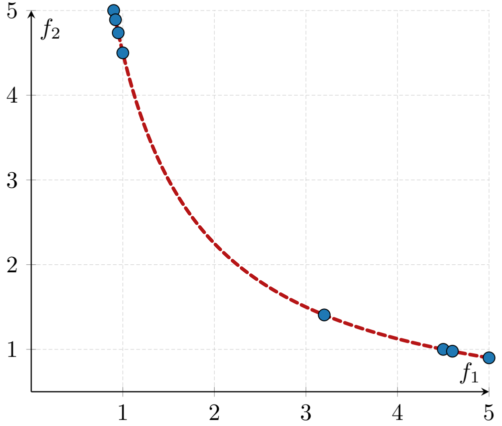
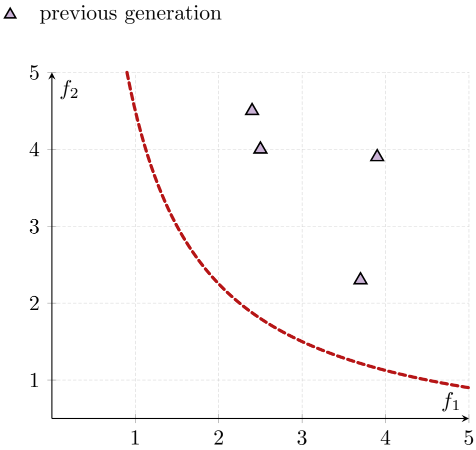


Deficiencies

- Often hypersensitive to weights w_i.
- Optimization has to run multiple times.

Our Approach[6]

1. There is A agents used at once, each has a unique set of weights $\boldsymbol{w}$.



[6] P. P. Junqueira, I. R. Meneghini, and F. G. Guimarães, "Multi-objective evolutionary algorithm based on decomposition with an external archive and local-neighborhood based adaptation of weights," *Swarm and Evolutionary Computation*, vol. 71, p. 101 079, 2022

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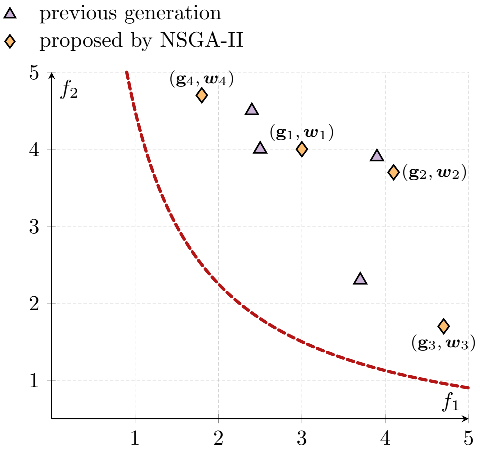

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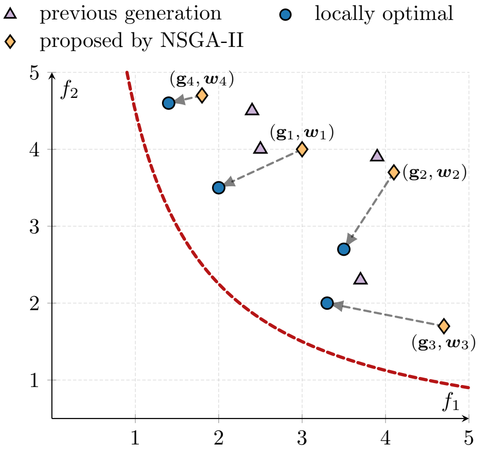

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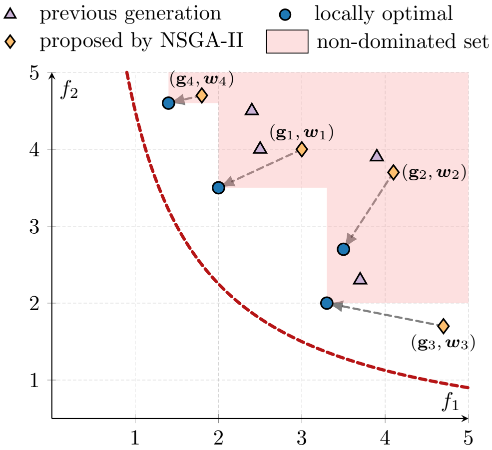

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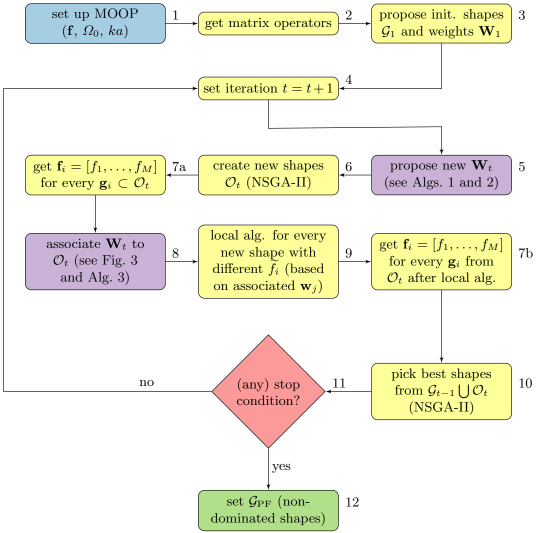
Algorithm[7]

- Global algorithm (NSGA-II) maintains robustness.
- Local algorithm (exact re-analysis) provides convergence.
- Adaptive assignment of weights yields Pareto efficiency.



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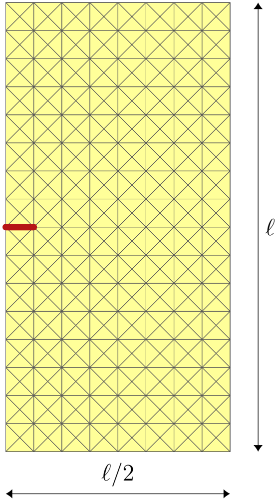
Example #1: Maximum Bandwidth × Electrical Size



Minimize {Q-factor, electrical size}

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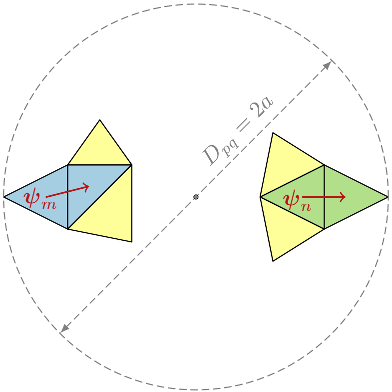

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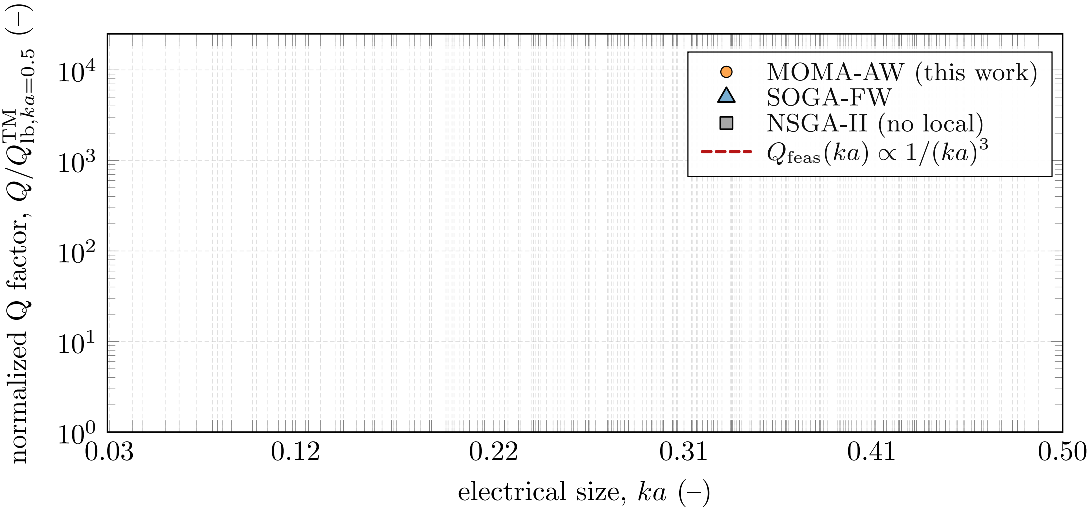
$$ka = \frac{1}{2} k \max\{\mathrm{diag}(\mathbf{g})\, \mathbf{D}\, \mathrm{diag}(\mathbf{g})\}.$$

Q-factor vs. Electrical Size: Pareto Efficiency



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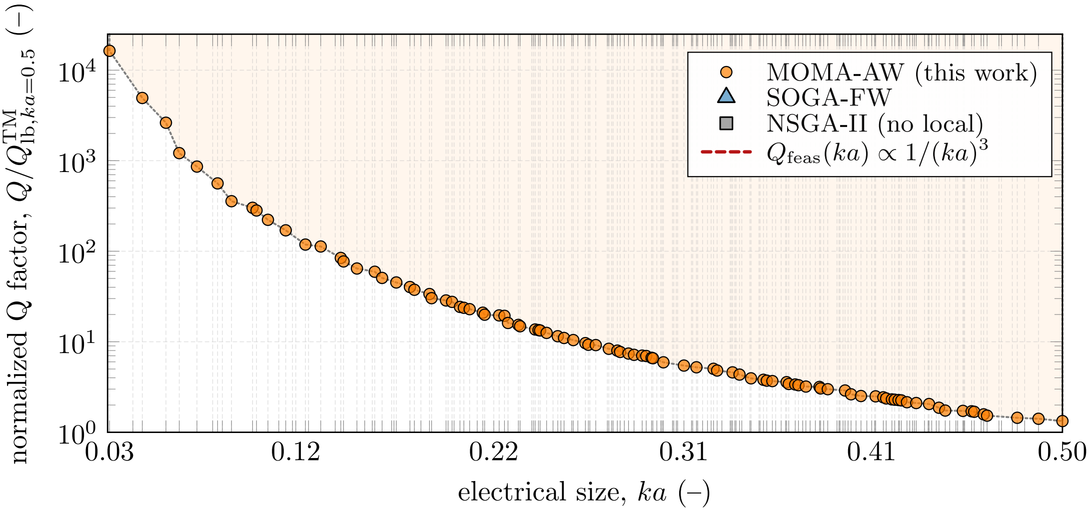

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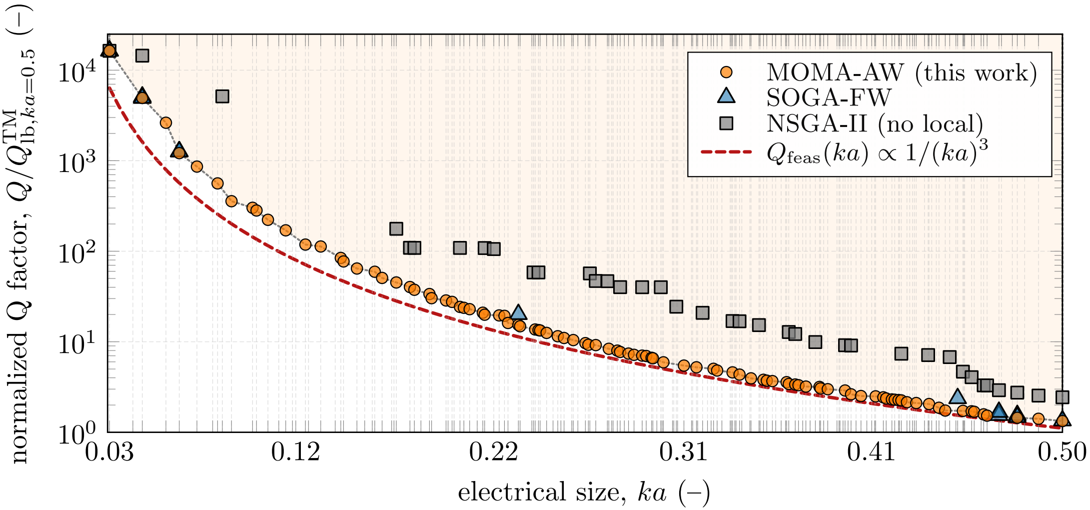

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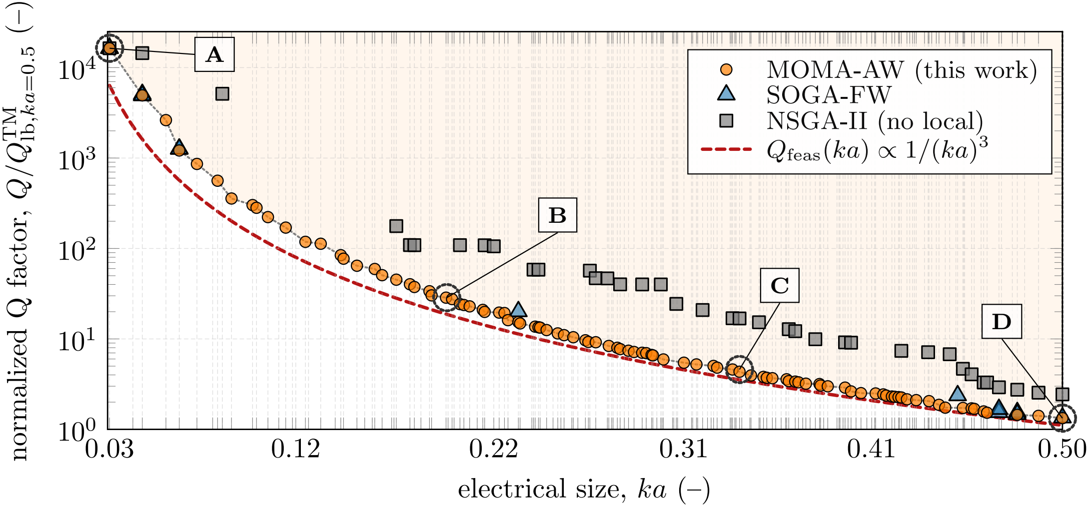


Q-factor vs. Electrical Size: Solutions

A

$$Q/Q^{\mathrm{TM}}_{\mathrm{lb},ka=0.5} = 16366$$
$$ka = 0.03$$

B

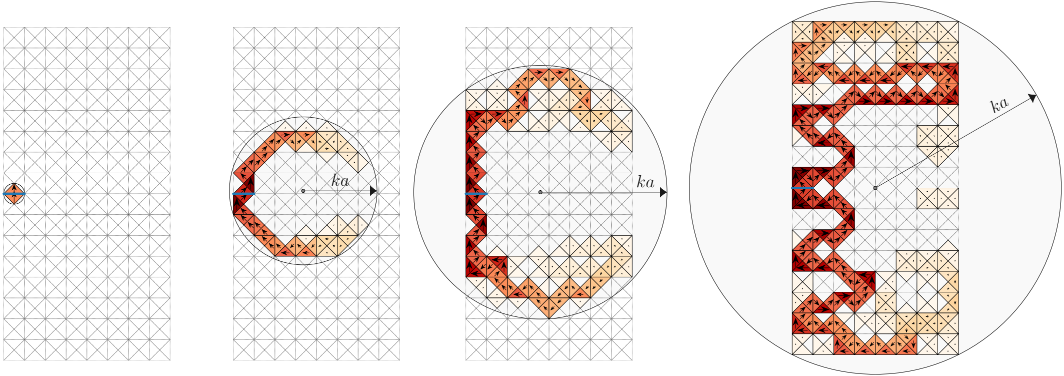
$$Q/Q^{\mathrm{TM}}_{\mathrm{lb},ka=0.5} = 28.6$$
$$ka = 0.19$$

C

$$Q/Q^{\mathrm{TM}}_{\mathrm{lb},ka=0.5} = 4.59$$
$$ka = 0.34$$

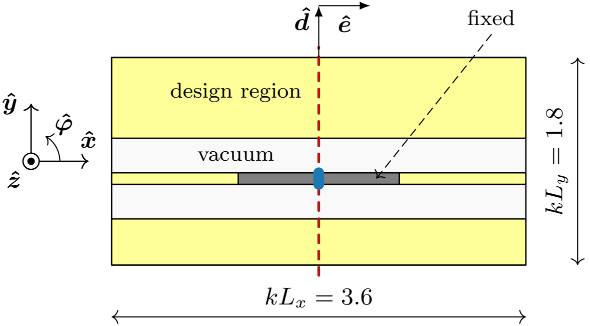
D

$$Q/Q^{\mathrm{TM}}_{\mathrm{lb},ka=0.50} = 1.33, \; ka = 0.50$$



- 44 millions of antennas evaluated in 35 minutes. . .

Example #2: Realized Gain(s) in Opposite Directions



Minimize realized gains

$$f = \{G_{\mathrm{r}}(\hat{\boldsymbol{d}}_1, \hat{\boldsymbol{e}}), G_{\mathrm{r}}(\hat{\boldsymbol{d}}_2, \hat{\boldsymbol{e}}), R\}$$

- Realized gain ($Z_0 = 50\,\Omega$):

$$G_{\mathrm{r}}(\hat{\boldsymbol{d}}, \hat{\boldsymbol{e}}) = \eta_{\mathrm{rad}} \left(1 - |\varGamma|^2\right) D(\hat{\boldsymbol{d}}, \hat{\boldsymbol{e}}),$$

η_{rad} radiation efficiency,
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D directivity.

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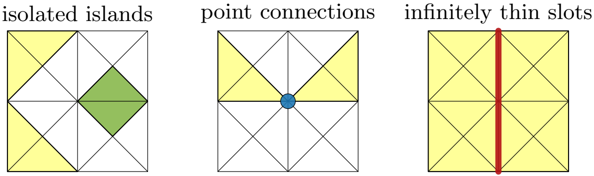
Shape regularity issues[8]:



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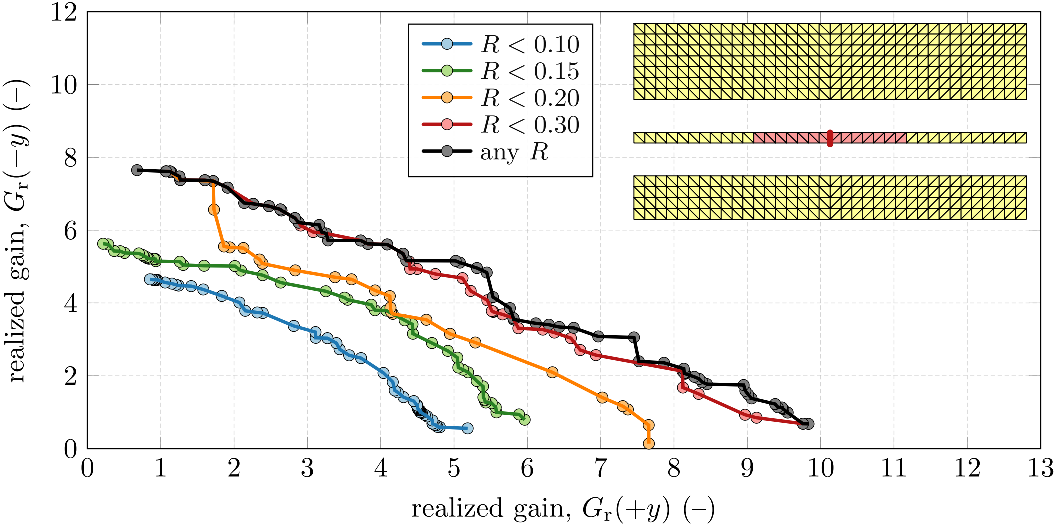
D directivity.

- Shape regularity

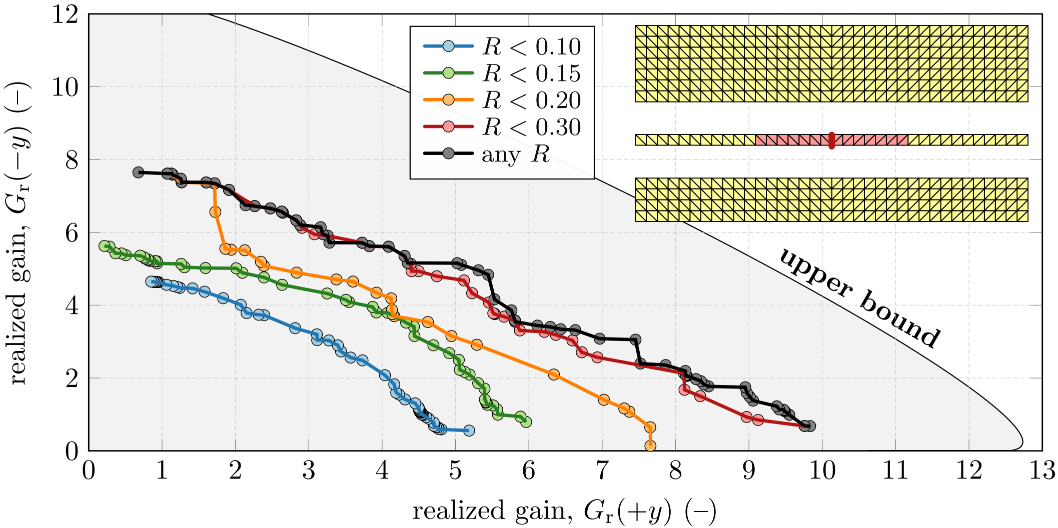
$$R = 0.15 \frac{A}{A_0} + 0.30\, h(\mathit{\Omega}).$$

[8]V. Neuman, M. Capek, and L. Jelinek, *Towards manufacturing-friendly shapes in discrete topology optimization*, 2025, https://arxiv.org/abs/2503.10133

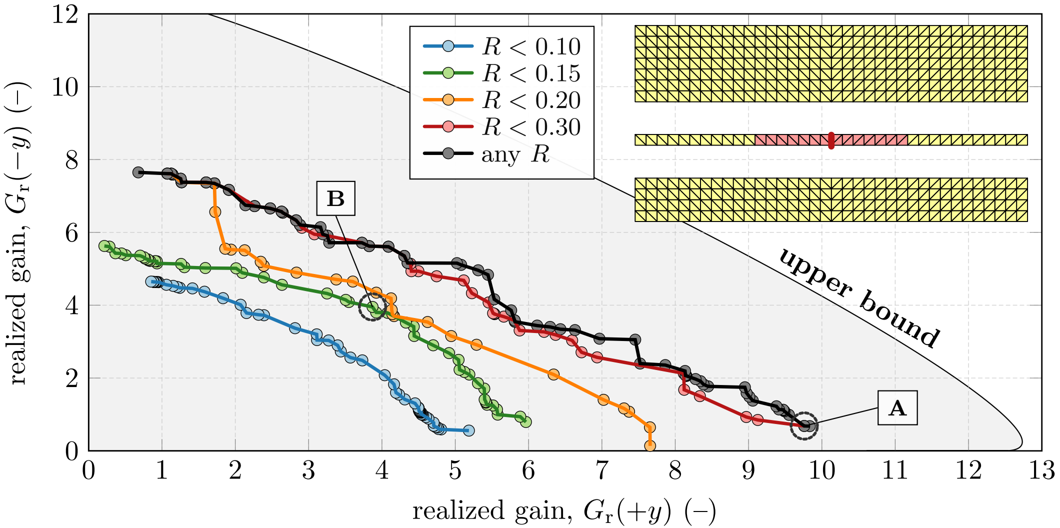
Realized Gain Trade-offs

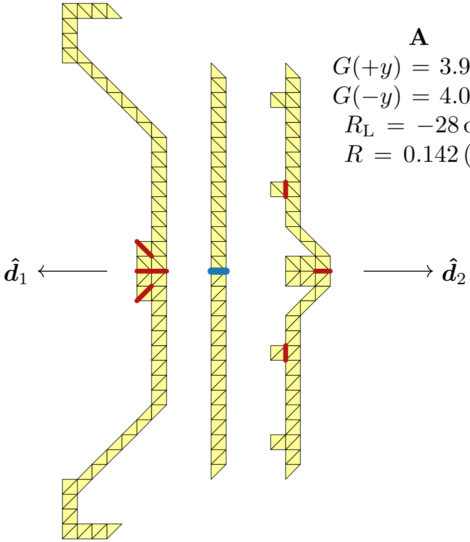


Realized Gain Trade-offs



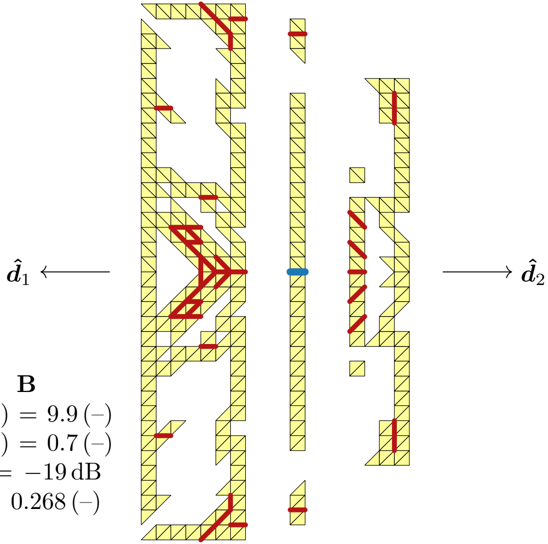
Realized Gain Trade-offs





A

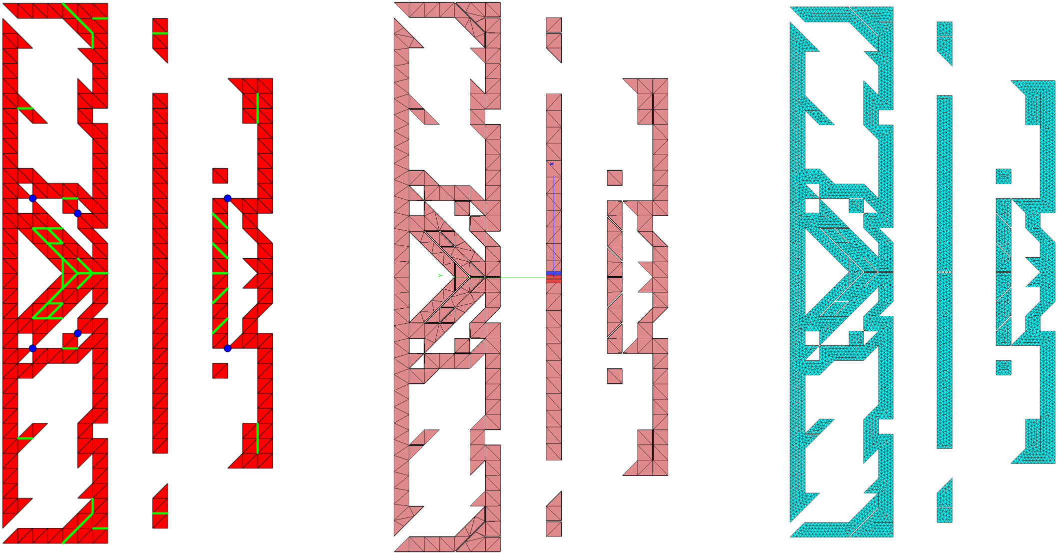
$$G(+y) = 3.9\,(-)$$
$$G(-y) = 4.0\,(-)$$
$$R_\mathrm{L} = -28\,\mathrm{dB}$$
$$R = 0.142\,(-)$$



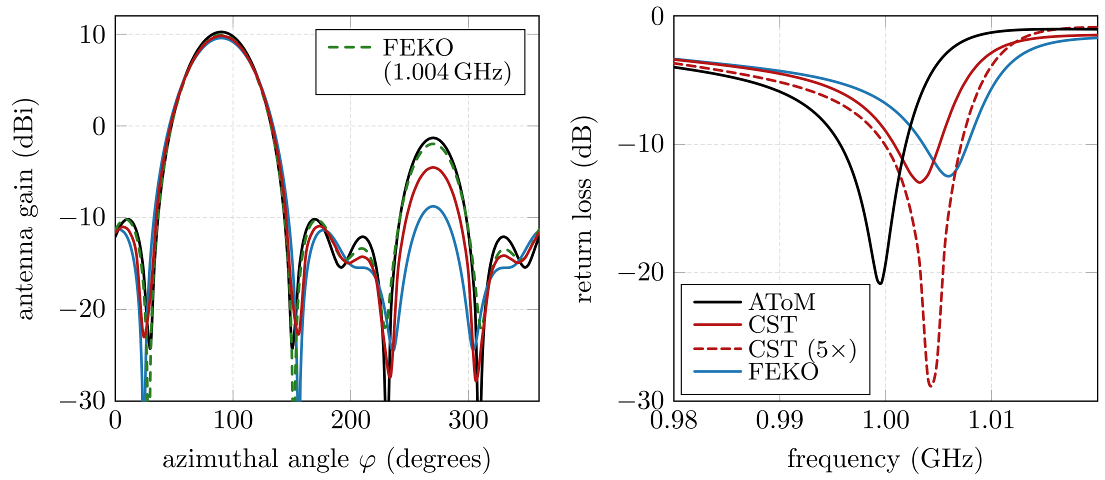
B

$$G(+y) = 9.9\,(-)$$
$$G(-y) = 0.7\,(-)$$
$$R_\mathrm{L} = -19\,\mathrm{dB}$$
$$R = 0.268\,(-)$$

- MATLAB script to export selected shapes to FEKO and CST.



Verification

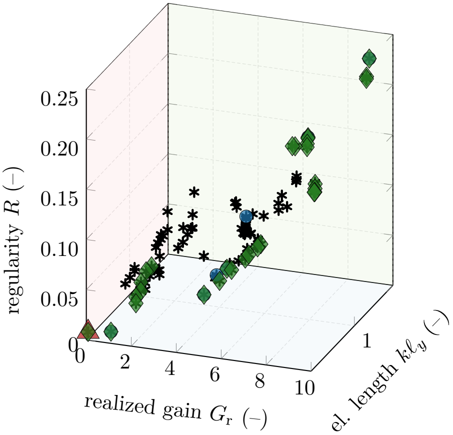


Concluding Remarks

Multi-Objective Memetic Algorithm with Adaptive Weights[9] (MOMA-AW)

What has been done…

1. Pareto efficiency.
2. Physics and geometry/topology optimized.
3. Great performance.



[9] P. Kadlec and M. Capek, *Multi-objective memetic algorithm with adaptive weights for inverse antenna design*, 2025, https://arxiv.org/abs/2409.14245.

Concluding Remarks

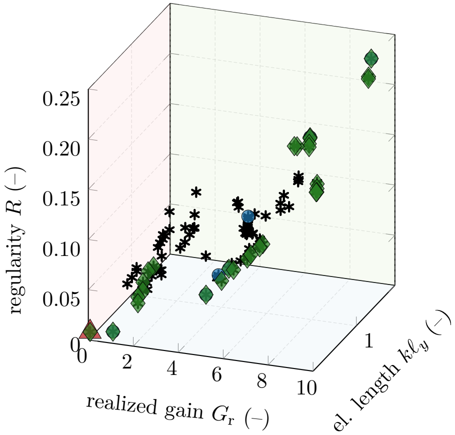
Multi-Objective Memetic Algorithm with Adaptive Weights[9] (MOMA-AW)

What has been done...

1. Pareto efficiency.
2. Physics and geometry/topology optimized.
3. Great performance.

What next?

1. Co-simulation with machine learning tools.
2. Principal component analysis.
3. Better maintenance of external archive.



[9] P. Kadlec and M. Capek, *Multi-objective memetic algorithm with adaptive weights for inverse antenna design*, 2025, https://arxiv.org/abs/2409.14245.

Questions?

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version 1.0
The presentation is available at ▶ capek.elmag.org

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Comparison With State-Of-The-Art Techniques