Upper Bound on Transducer and Radiation Efficiencies of a Multi-port Antenna Array

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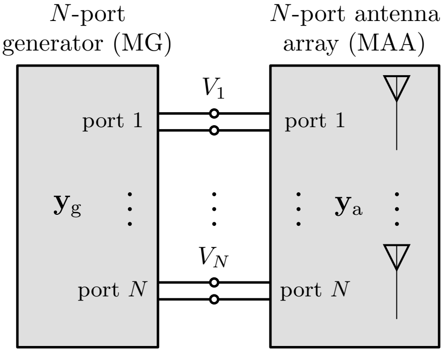
The goal is to determine the performance limits of multi-port antennas (MAA) in terms of efficiency[1].



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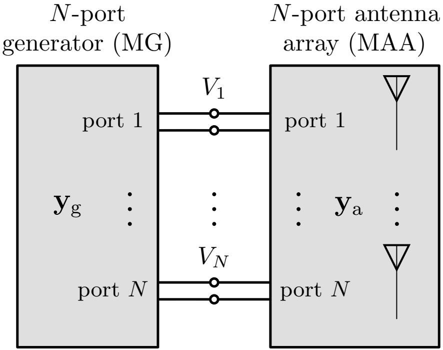
- How to define (total) efficiency?
- How to cope with (ohmic) losses?
- How much do we know about excitation?



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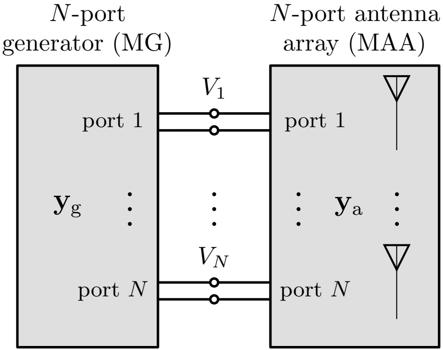

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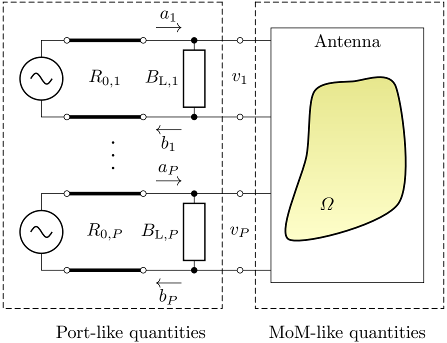
Total Active Reflection Coefficient[2] – Definition

$$\Gamma^{\mathrm{t}} = \sqrt{1 - \frac{P_{\mathrm{rad}}}{P_{\mathrm{in}}}}$$



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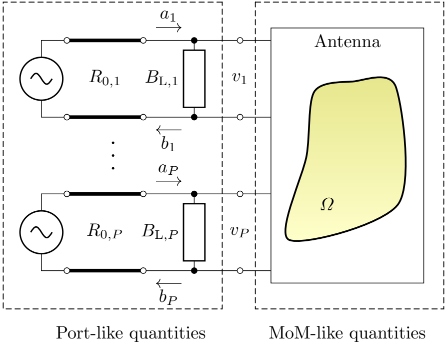
$$\eta_{\mathrm{rad}} = \frac{P_{\mathrm{rad}}}{P_{\mathrm{rad}} + P_{\mathrm{lost}}} \qquad \eta_{\mathrm{match}} = \frac{P_{\mathrm{rad}} + P_{\mathrm{lost}}}{P_{\mathrm{in}}}$$

$$\eta_{\mathrm{tot}} = \eta_{\mathrm{rad}}\eta_{\mathrm{match}} = 1 - \left(\Gamma^{\mathrm{t}}\right)^2$$



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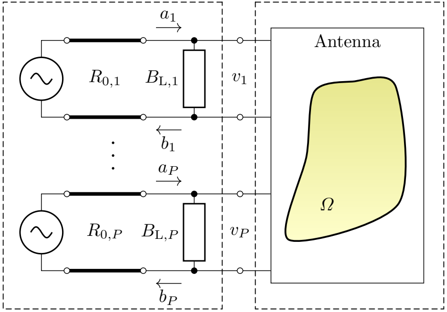

Port-like quantities MoM-like quantities

- Maximizing total efficiency → minimizing TARC.

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Total Efficiency – Optimization

Total efficiency to be maximized (TARC is to be minimized):

$$\begin{aligned} \text{maximize} \quad & P_{\mathrm{rad}} \\ \text{subject to} \quad & P_{\mathrm{in}} = 1 \end{aligned}$$

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- How to evaluate P_{in} and P_{rad}?

Power Terms

Power waves

Total (input) power

$$P_{\mathrm{in}} = \frac{1}{2}\mathbf{a}^{\mathrm{H}}\mathbf{a}.$$

Radiated power

$$P_{\mathrm{rad}} = \frac{1}{2}\left(\mathbf{a}^{\mathrm{H}}\mathbf{a} - \mathbf{b}^{\mathrm{H}}\mathbf{b}\right).$$

Reflected power

$$P_{\mathrm{refl}} = \frac{1}{2}\mathbf{b}^{\mathrm{H}}\mathbf{b}.$$

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Method of moments

Total (input) power

$$P_{\mathrm{in}} = P_{\mathrm{rad}} + P_{\mathrm{lost}} + P_{\mathrm{refl}}.$$

Radiated power (always valid)

$$P_{\mathrm{rad}} = \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_0\mathbf{I}.$$

Ohmic losses

$$P_{\mathrm{lost}} = \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_\rho\mathbf{I}.$$

Power Terms in Method of Moments

MoM representation

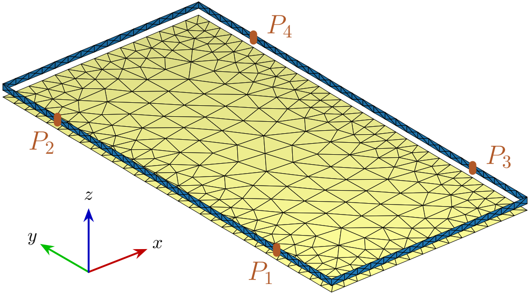
$$\boldsymbol{J}(\boldsymbol{r}) \approx \sum_{n=1}^{N} I_n \boldsymbol{\psi}_n(\boldsymbol{r})$$

where $\mathbf{I} = [I_n] \in \mathbb{C}^{N \times 1}$ are expansion coefficients gives

$$\mathbf{V} = \mathbf{Z}\mathbf{I}$$

with impedance matrix $\mathbf{Z} \in \mathbb{C}^{N \times N}$ being

$$\mathbf{Z} = \mathbf{R}_0 + \mathbf{R}_\rho + \mathrm{j}\mathbf{X}_0.$$



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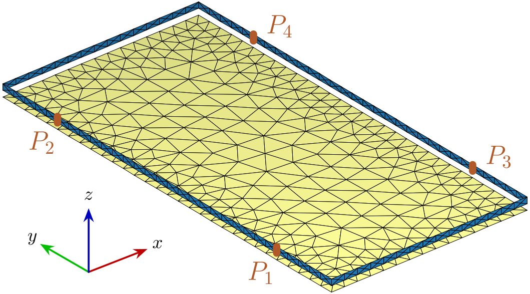

- We need to express all parameters in terms of port quantities.

Port-mode Representation

Antenna MoM

$$\mathbf{I} = \mathbf{Y}\mathbf{V}$$

Port MoM

$$\mathbf{i} = \mathbf{y}\mathbf{v}$$

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Port current

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$\mathbf{C} \in \{0,1\}^{N\times P}$ is sparse indexing matrix, $\mathbf{C}^{\mathrm{H}}\mathbf{C} = \mathbf{1} \in \mathbb{R}^{P\times P}$ with

$$C_{np} = \begin{cases} 1 & p\text{-th port is placed at } n\text{-th position,} \\ 0 & \text{otherwise.} \end{cases}$$

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Port admittance matrix

$$\mathbf{y} = \mathbf{C}^{\mathrm{H}}\mathbf{Y}\mathbf{C}$$

Port-mode Quantities

The transformation between current and port quantities is generalized as

$$\mathbf{I}^{\mathrm{H}}\mathbf{M}\mathbf{I} = \mathbf{v}^{\mathrm{H}}\mathbf{m}\mathbf{v}$$

for arbitrary $\mathbf{M}$

$$\mathbf{m} = \mathbf{C}^{\mathrm{H}}\mathbf{Y}^{\mathrm{H}}\mathbf{M}\mathbf{Y}\mathbf{C}.$$

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[3] D. M. Pozar, *Microwave Enginnering*, 4th. Wiley, 2011

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Port-mode equivalents:

$$P_{\mathrm{rad}} = \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_0\mathbf{I} = \frac{1}{2}\mathbf{v}^{\mathrm{H}}\mathbf{g}_0\mathbf{v}$$

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$$\left(\mathbf{Z} = \mathbf{R}_0 + \mathbf{R}_\rho + \mathrm{j}\mathbf{X}_0\right)$$

Power waves[3] $\left(\boldsymbol{\Lambda} = \mathrm{diag}\left(\sqrt{R_{0,p}}\right), \, \mathbf{y}_{\mathrm{L}} = \mathrm{diag}\left(\mathrm{j}B_{\mathrm{L},p}\right)\right)$

$$\mathbf{a} = \frac{1}{2}\left(\boldsymbol{\Lambda}^{-1}\mathbf{v} + \boldsymbol{\Lambda}\mathbf{i}\right) = \mathbf{k}_{\mathrm{i}}\mathbf{v}$$

$$\mathbf{b} = \frac{1}{2}\left(\boldsymbol{\Lambda}^{-1}\mathbf{v} - \boldsymbol{\Lambda}\mathbf{i}\right) = \mathbf{k}_{\mathrm{r}}\mathbf{v}$$

$$\left(\mathbf{i} = \left(\mathbf{y} + \mathbf{y}_{\mathrm{L}}\right)\mathbf{v}\right)$$

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Total Efficiency (TARC) Optimization

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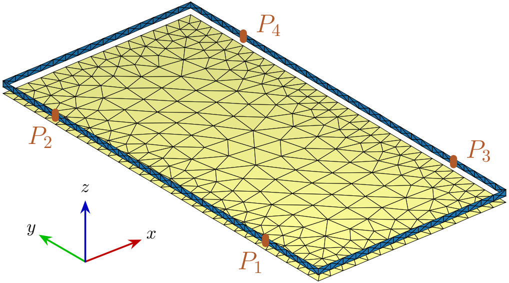

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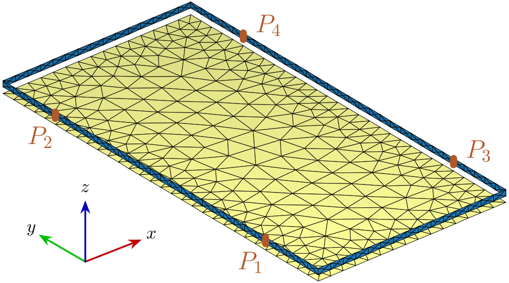

Various levels of complexity[4]:

- optimal excitation of ports,
- optimal placement of ports,
- (optimal) number of ports,
- optimal matching circuitry.

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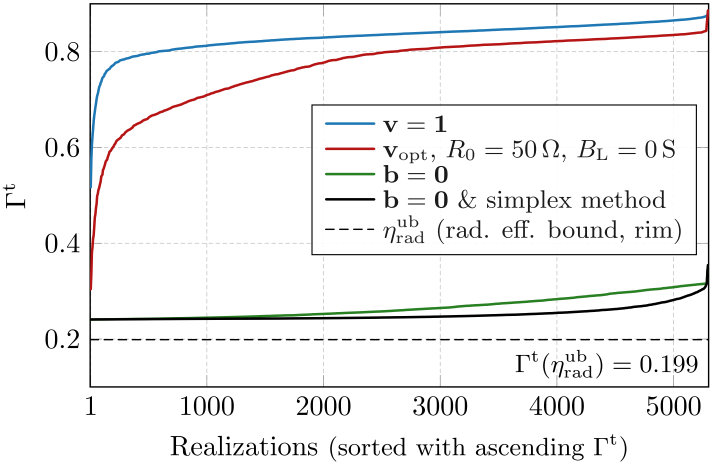
Realistic Example – Metallic Rim



Deficiencies (for MIMO)

Knowledge of a multi-port generator

1. The approach assumes that MG is known and all its ports are decoupled.
    - This might not be the case in practice.
    - Use available power P_{ava}.

$$P_{\text{in}} \longrightarrow P_{\text{ava}}$$

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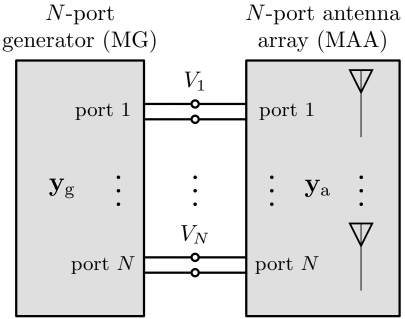
Control of port voltages

2. The approach assumes that the optimal voltages are always impressed.
   - This is not the case of MIMO antennas.
   - Take care of minimal values as well.

$$\eta_{\mathrm{min}} \leq \eta(\mathbf{v}) \leq \eta_{\mathrm{max}}$$

Available power[5]

Maximum cycle mean power that can be drawn from the generator by an arbitrary LTI and passive load.



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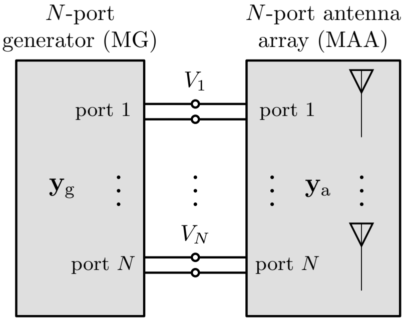
$$P_{\mathrm{ava}} = \frac{1}{2}\mathbf{v}^{\mathrm{H}}\mathbf{g}_{\mathrm{ava}}\mathbf{v}$$

$$\mathbf{g}_{\mathrm{ava}} = \frac{1}{2}\left(\mathbf{y}_{\mathrm{a}} + \mathbf{y}_{\mathrm{g}}\right)^{\mathrm{H}}\left(\mathbf{y}_{\mathrm{g}} + \mathbf{y}_{\mathrm{g}}^{\mathrm{H}}\right)^{-1}\left(\mathbf{y}_{\mathrm{a}} + \mathbf{y}_{\mathrm{g}}\right)$$



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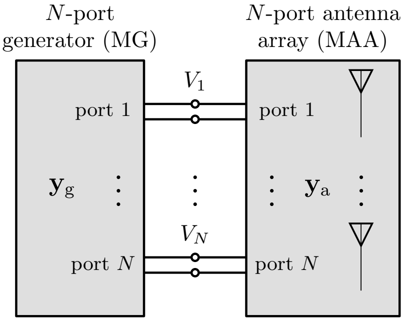
- Suitable in cases when the internal structure of the generator is complex or not known.



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Approach #2 – Transducer Efficiency

$$\begin{aligned} &\text{maximize/minimize} \quad && \mathbf{v}^{\mathrm{H}} \mathbf{g}_0 \mathbf{v} \\ &\text{subject to} && \mathbf{v}^{\mathrm{H}} \mathbf{g}_{\mathrm{ava}} \mathbf{v} = 1 \end{aligned}$$

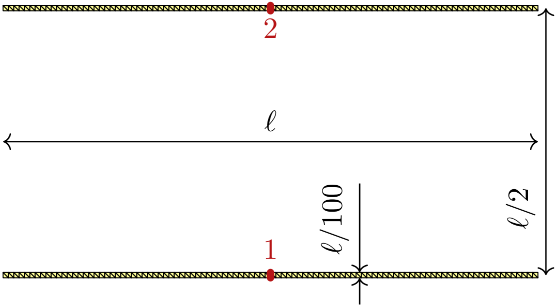
solved as

$$\mathbf{g}_0 \mathbf{v}_n = \eta_{\mathrm{T},n} \mathbf{g}_{\mathrm{ava}} \mathbf{v}_n$$

- Take both the maximum and minimum value.

Example

- Two dipoles, material: copper,
- $\ell = 187.5\,\mathrm{mm}$, $f \in [400, 1200]\,\mathrm{MHz}$,
- two uncoupled generators, $50\,\Omega$ TML,
- two Mini-Circuits NCS1.5-232+ RF baluns.



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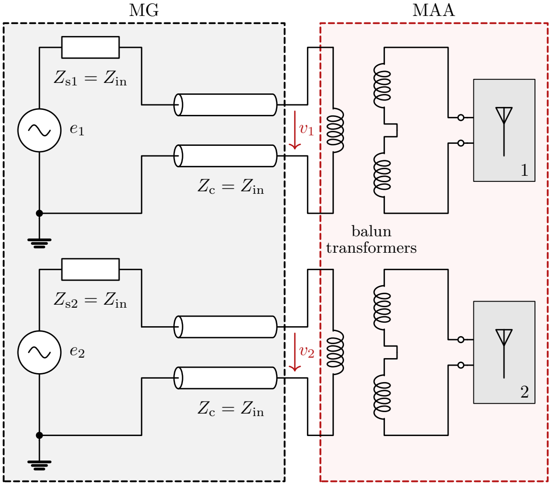

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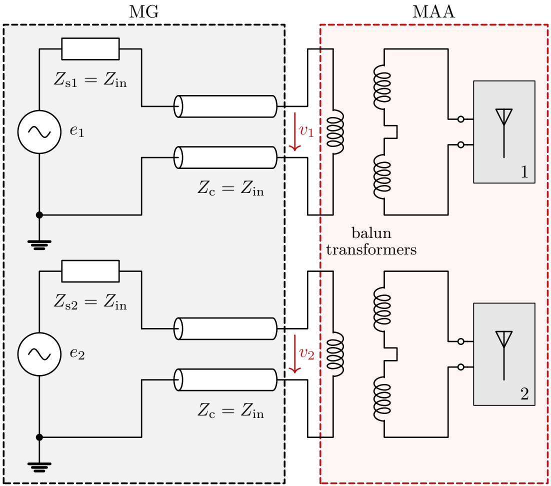

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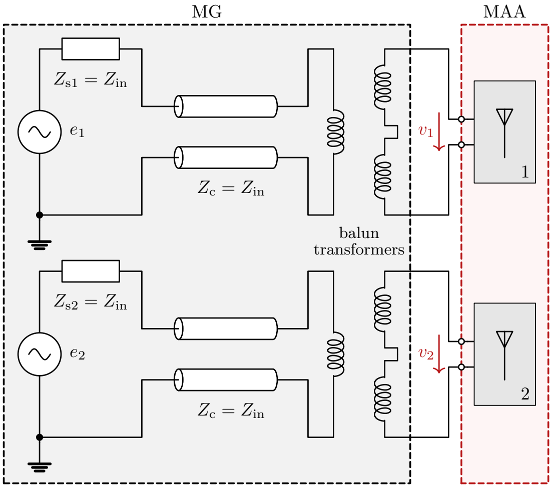

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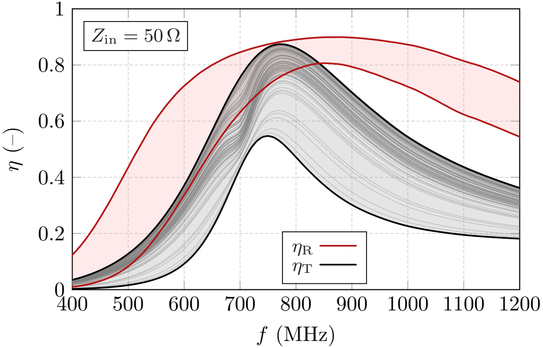

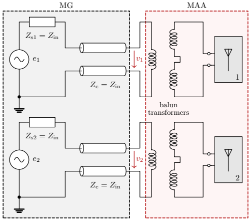

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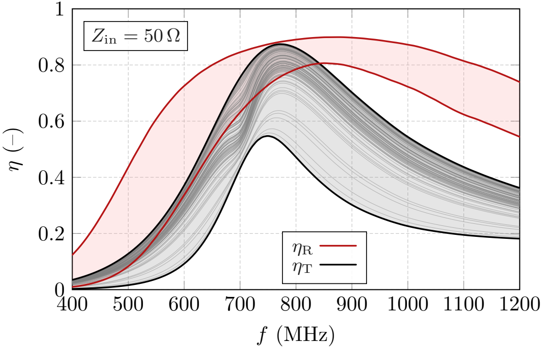

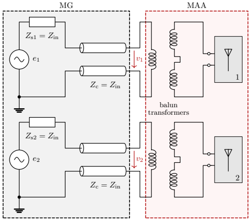

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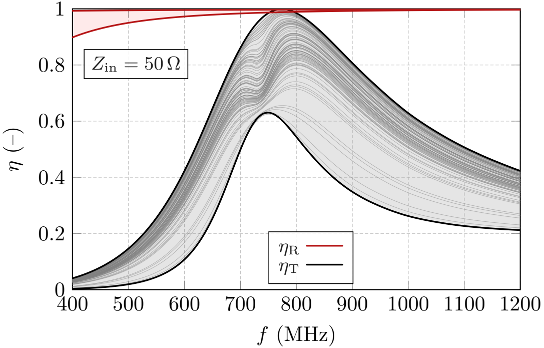

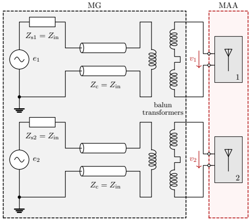

Conclusion

- Performance estimates depend on[6]
    - knowledge about the system,
    - type of the system to be designed.

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Conclusion

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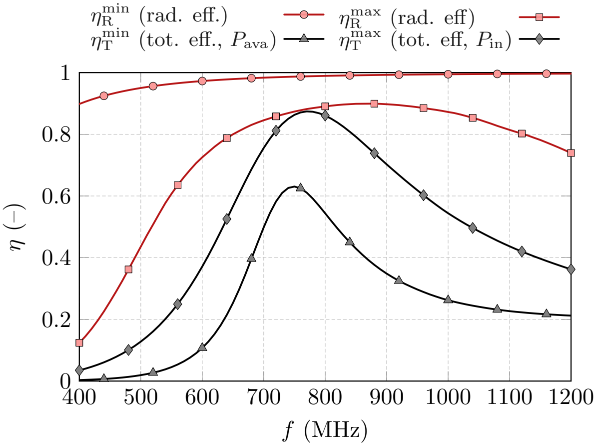
  - knowledge about the system,
  - type of the system to be designed.
- Use
  - radiation × total efficiency,
  - maximum × minimum estimates,
  - input × available power.

1. MAA ($\eta_\mathrm{R}^\mathrm{min}$)
2. $\mathbf{y}_\mathrm{g}$ ($\eta_\mathrm{T}^\mathrm{min}$, P_ava)
3. set $\mathbf{v}$ ($\eta_\mathrm{T}^\mathrm{max}$, P_ava)
4. MG ($\eta_\mathrm{R}^\mathrm{max}$)
5. set $\mathbf{v}$ & matching ($\eta_\mathrm{T}^\mathrm{max}$, P_in)



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[6] M. Capek and L. Jelinek, “Transducer and radiation efficiency figures of a multiport antenna array,” *IEEE Trans. Antennas Propag.*, vol. 71, no. 7, pp. 6132–6137, Jul. 2023

Questions?

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