Unified Approach to Characteristic Modes

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December 7, 2021
IEEE AP-S/URSI 2021
Singapore
Dominant characteristic mode of a fractal structure (illustrative picture).
“There’s nothing more practical than a good theory” — K. Lewin

(a) Daviu et al., UWB ant, 2010, (b) Li et al., cellphone chassis ant., 2012, (c) Ethier and McNamara, shape synthesis, 2014, (d) Dicandia et al., null-steering, 2016, (e) Yang and Adams, MIMO ant. synthesis, 2016.
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\textbf{328 WAVEGUIDE JUNCTIONS WITH SEVERAL ARMS}  

The eigenvalue solutions\textsuperscript{1} are convenient for the calculation of scattering. Let

\[ S a_k = \lambda_k a_k, \]  \hspace{1cm} (94)

where the $a_k$ are a complete, orthogonal, and real set of eigenvectors. The column vectors $a_k$ are normalized in such a way that

\[ a_k^* a_k = 1. \]  \hspace{1cm} (95)

Thus for each of these eigensolutions, the power radiated by the antenna is

\[ \frac{1}{2} a_k^* a_k^{(1)} = \frac{1}{2} a_k^2, \]  \hspace{1cm} (96)

(Short) Historical Overview

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1968 Rigorously introduced by Garbacz² as diagonalization of the perturbation operator.


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1968 Rigorously introduced by Garbacz\textsuperscript{2} as diagonalization of the perturbation operator.

1971 Generalized by Harrington and Mautz\textsuperscript{3} for arbitrarily shaped antennas.


\textsuperscript{2}R. J. Garbacz, “A generalized expansion for radiated and scattered fields,” Ph.D. dissertation, Department of Electrical Engineering, Ohio State University, 1968

Characteristic Modes

Scattering operator $S$

$$f_0 = S f_i$$

$$S f_{o,n} = s_n f_{o,n}$$
Scattering operator $S$

$$f_o = S f_i$$

$$S f_{o,n} = s_n f_{o,n}$$

Transition operator $T$

$$f_o = T a$$

$$T f_{o,n} = t_n f_{o,n}$$
Scattering operator $S$

$$f_o = S f_i$$

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Transition operator $T$

$$f_o = T a$$

$$T f_{o,n} = t_n f_{o,n}$$

Impedance operator $Z$

$$-\hat{n} \times \hat{n} \times E_i = Z J$$

$$Z J_n = (1 + j \lambda_n) R J_n$$
Characteristic Modes

\[ f_o = S f_i \]
\[ f_o = T a \]
\[ S f_{o,n} = s_n f_{o,n} \]
\[ T f_{o,n} = t_n f_{o,n} \]

Scattering operator \( S \)
Transition operator \( T \)
Impedance operator \( Z \)

\[ -\hat{n} \times \hat{n} \times E_i = Z J \]
\[ Z J_n = (1 + j \lambda_n) \mathcal{R} J_n \]

scattering problem
antenna problem
Properties of $T$ and $Z$ Decomposition...

\[ ZI_n = (1 + j\lambda_n)RI_n \]

\[ Tf_{o,n} = -\frac{1}{1 + j\lambda_n}f_{o,n} \]

- Mode 1 (cap.)
- Mode 2 (cap.)
- Mode 3 (ind.)
- Mode 4 (cap.)
- Mode 5 (ind.)

\[ P_{r,1} = 1.000 \text{ W} \]
\[ P_{r,2} = 0.586 \text{ W} \]
\[ P_{r,3} = 0.050 \text{ W} \]
\[ P_{r,4} = 0.044 \text{ W} \]
\[ P_{r,5} = 0.001 \text{ W} \]
Scattering and Impedance Definitions

Properties of $T$ and $Z$ Decomposition...

\[ ZI_n = (1 + j\lambda_n)RI_n \]

\[ Tf_{o,n} = -\frac{1}{1 + j\lambda_{o,n}}f_{o,n} \]

Prot

- PEC formulation well implemented.
- Easy to evaluate for complex shapes.
- Current densities directly accessible.

Pros

- Unique definition.
- Arbitrary (inhomogeneous) materials.
- Allows for powerful tracking.
- Independent of numerical method used.
Properties of $\mathbf{T}$ and $\mathbf{Z}$ Decomposition...

$$ZI_n = (1 + j\lambda_n)RI_n$$

$$Tf_{o,n} = -\frac{1}{1 + j\lambda_{o,n}}f_{o,n}$$

**Cons**

- Spurious modes/internal resonances.
- Ambiguous definition for material bodies.
- Based on method of moments.
- Large matrix to be decomposed for a complicated shape.

**Cons**

- Problematic for complex shapes.
- Current densities unknown.
Properties of $T$ and $Z$ Decomposition...

\[
ZI_n = (1 + j\lambda_n)RI_n
\]

\[
Tf_{o,n} = -\frac{1}{1 + j\lambda_n}f_{o,n}
\]

What about to unify both definitions and take the best from both?
Initial Formulas

Method of moments:

\[ E^S (r) = -jkZ_0 \int_\Omega G (r, r') \cdot J (r') \, dV' \]

\[ J (r) \approx \sum_p I_p \psi_p (r) \]

\[ ZI = V \]
Initial Formulas

Method of moments:

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\[ J(r) \approx \sum_p I_p \psi_p(r) \]

\[ ZI = V \]

Transition matrix:

\[ T_a = f \]

\[ E^i(r) \approx k \sqrt{Z_0} \sum_\alpha a_\alpha u^{(1)}_\alpha(kr) \]

\[ E^s(r) \approx k \sqrt{Z_0} \sum_\alpha f_\alpha u^{(4)}_\alpha(kr) \]
Initial Formulas

Method of moments:

\[ E^s (r) = -j k Z_0 \int_{\Omega} G (r, r') \cdot J (r') \, dV' \]

\[ J (r) \approx \sum_p I_p \psi_p (r) \]

\[ Z I = V \]

Transition matrix:

\[ T a = f \]

\[ E^i (r) \approx k \sqrt{Z_0} \sum_{\alpha} a_{\alpha} u^{(1)}_{\alpha} (k r) \]

\[ E^s (r) \approx k \sqrt{Z_0} \sum_{\alpha} f_{\alpha} u^{(4)}_{\alpha} (k r) \]

Decomposition of Green’s Dyadics

\[ G (r_1, r_2) = -j k \sum_{\alpha} u^{(4)}_{\alpha} (k r_1) u^{(1)}_{\alpha} (k r_2) \]
Projections Between Bases – From $Z$ To $T$

Scattered field

$$k \sqrt{Z_0} \sum_{\alpha} f_{\alpha} u_{\alpha}^{(4)} (kr) = -j k Z_0 \int_{\Omega} (-jk) \sum_{\alpha} u_{\alpha}^{(4)} (r) u_{\alpha}^{(1)} (r') \cdot \sum_{p} I_p \psi_p (r') \, dV$$
Scattered field

\[ k\sqrt{Z_0} \sum_{\alpha} f_\alpha u^{(4)}_\alpha (kr) = -jkZ_0 \int_{\Omega} \left( -jk \right) \sum_{\alpha} u^{(4)}_\alpha (r) u^{(1)}_\alpha (r') \cdot \sum_p I_p \psi_p (r') \, dV \]

Testing from left with \( u^{(4)}_\beta (kr) \) (utilize orthogonality of spherical waves):

\[ f_\alpha = -\sum_p I_p k\sqrt{Z_0} \int_{\Omega} u^{(1)}_\alpha (r') \cdot \psi_p (r') \, dV \]
Scattered field

\[ k \sqrt{Z_0} \sum_{\alpha} f_{\alpha} u^{(4)}_{\alpha}(kr) = -jkZ_0 \int_{\Omega} (-jk) \sum_{\alpha} u^{(4)}_{\alpha}(r) u^{(1)}_{\alpha}(r') \cdot \sum_{p} I_p \psi_p(r') \, dV \]

Testing from left with \( u^{(4)}_{\beta}(kr) \) (utilize orthogonality of spherical waves):

\[ f_{\alpha} = -\sum_{p} I_p \, k \sqrt{Z_0} \int_{\Omega} u^{(1)}_{\alpha}(r') \cdot \psi_p(r') \, dV \Rightarrow f = -UI \]
Scattered field

\[ k \sqrt{Z_0} \sum_{\alpha} f_{\alpha} u_{\alpha}^{(4)} (kr) = -jkZ_0 \int_{\Omega} (-jk) \sum_{\alpha} u_{\alpha}^{(4)} (r) u_{\alpha}^{(1)} (r') \cdot \sum_{p} I_p \psi_p (r') \, dV \]

Testing from left with \( u_{\beta}^{(4)} (kr) \) (utilize orthogonality of spherical waves):

\[ f_{\alpha} = -\sum_{p} I_p k \sqrt{Z_0} \int_{\Omega} u_{\alpha}^{(1)} (r') \cdot \psi_p (r') \, dV \implies f = -UI \]
Scattered field

\[ k \sqrt{Z_0} \sum_{\alpha} f_{\alpha} u_{\alpha}^{(4)} (k r) = -j k Z_0 \int_{\Omega} (-j k) \sum_{\alpha} u_{\alpha}^{(4)} (r) u_{\alpha}^{(1)} (r') \cdot \sum_{p} I_p \psi_p (r') \ dV \]

Testing from left with \( u_{\beta}^{(4)} (k r) \) (utilize orthogonality of spherical waves):

\[ f_{\alpha} = - \sum_{p} I_p k \sqrt{Z_0} \int_{\Omega} u_{\alpha}^{(1)} (r') \cdot \psi_p (r') \ dV \Rightarrow f = -UI \]

Incident field

\[ V_p = \int_{\Omega} \psi_p (r) \cdot E_i (r) \ dV = k \sqrt{Z_0} \int_{\Omega} \psi_p (r) \cdot \sum_{\alpha} a_{\alpha} u_{\alpha}^{(1)} (k r) \ dV \]
Scattered field

\[ k\sqrt{Z_0} \sum_\alpha f_\alpha u^{(4)}_\alpha (kr) = -jkZ_0 \int_\Omega (-jk) \sum_\alpha u^{(4)}_\alpha (r) u^{(1)}_\alpha (r') \cdot \sum_p I_p \psi_p (r') \, dV \]

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\[ V_p = \sum_\alpha a_\alpha k\sqrt{Z_0} \int_\Omega \psi_p (r) \cdot u^{(1)}_\alpha (kr) \, dV \]
Scattered field

\[ k\sqrt{Z_0} \sum_{\alpha} f_\alpha u_\alpha^{(4)}(kr) = -jkZ_0 \int_{\Omega} (-jk) \sum_{\alpha} u_\alpha^{(4)}(r) u_\alpha^{(1)}(r') \cdot \sum_{p} I_p \psi_p(r') \, dV \]

Testing from left with \( u_\beta^{(4)}(kr) \) (utilize orthogonality of spherical waves):

\[ f_\alpha = -\sum_{p} I_p k\sqrt{Z_0} \int_{\Omega} u_\alpha^{(1)}(r') \cdot \psi_p(r') \, dV \quad \Rightarrow \quad f = -UI \]

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\[ V_p = \sum_{\alpha} a_\alpha k\sqrt{Z_0} \int_{\Omega} \psi_p(r) \cdot u_\alpha^{(1)}(kr) \, dV \quad \Rightarrow \quad V = U^T a \]
Projections Between Bases – From $Z$ To $T$

Scattered field

$$k\sqrt{Z_0} \sum_{\alpha} f_\alpha \mathbf{u}_\alpha^{(4)} (kr) = -jkZ_0 \int_{\Omega} (-jk) \sum_{\alpha} \mathbf{u}_\alpha^{(4)} (r) \mathbf{u}_\alpha^{(1)} (r') \cdot \sum_{p} I_p \psi_p (r') \, dV$$

Testing from left with $\mathbf{u}_\beta^{(4)} (kr)$ (utilize orthogonality of spherical waves):

$$f_\alpha = -\sum_{p} I_p k\sqrt{Z_0} \int_{\Omega} \mathbf{u}_\alpha^{(1)} (r') \cdot \psi_p (r') \, dV \quad \Rightarrow \quad f = -UI$$

Incident field

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$$V_p = \sum_{\alpha} a_\alpha k\sqrt{Z_0} \int_{\Omega} \psi_p (r) \cdot \mathbf{u}_\alpha^{(1)} (kr) \, dV \quad \Rightarrow \quad V = U^T a$$
Evaluation of T-matrix From Impedance Matrix

\[ f = -UI \]

Projecting matrix \( U \) between regular waves and local basis:

\[ U_{\alpha p} = k \sqrt{Z_0} \int_{\Omega} u_{\alpha}^{(1)}(kr) \cdot \psi_p(r) \, dV \]
Evaluation of T-matrix From Impedance Matrix

\[ f = -UI = -UZ^{-1}V \]

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Evaluation of T-matrix From Impedance Matrix

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Evaluation of T-matrix From Impedance Matrix

\[ f = -UI = -UZ^{-1}V = -UZ^{-1}U^Ta = Ta \]

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Projecting matrix \( U \) between regular waves and local basis:

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The work-flow is as follows:

\[
Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow Tf_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n
\]

- Valid for arbitrary MoM code (matrix \( U \) may be a bit more complicated).
- Calculation of \( Z \) and \( U \) matrices is sufficient to get transition matrix \( T \).
**Properties**

\[ Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow T f_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n \]

1. Only external resonances sought for.
Properties

\[ Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow T f_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n \]

1. Only external resonances sought for.
2. Straightforward interpretation.

- \( \lambda_n \) is the eigenvalue for \( T \) and \( Z \).
- \( f_n \) is the eigenvector for \( T \) and \( Z \).
- \( I_n \) is the reaction vector for \( T \) and \( Z \).

---

**Graphical Representation:**

- **TM/TE modes**
  - Logarithmic scale for \( |\lambda_n| \)
  - Spheres and arrows indicate TM and TE modes.

- **ind./cap. modes**
  - Logarithmic scale for \( |\lambda_n| \)
  - Grid and arrows indicate capacitive and inductive modes.

- \( \circ \) decomposition of \( T \)
- \( \times \) decomposition of \( Z \)
Properties

\[ Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow T f_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n \]

1. Only external resonances sought for.
2. Straightforward interpretation.
3. No spurious modes, etc.

- Decomposition of \( T \)
- Decomposition of \( Z \)
Unification and Properties

Properties

\[ Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow T f_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n \]

1. Only external resonances sought for.
2. Straightforward interpretation.
3. No spurious modes, etc.
4. Compact representation.

\[ \log_{10} |\lambda_n| \]

- TM/TE modes
- cap.
- ind.

\[ \text{decomposition of } T \]
\[ \text{decomposition of } Z \]
**Unification and Properties**

\[
Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow T f_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n
\]

1. Only external resonances sought for.
2. Straightforward interpretation.
3. No spurious modes, etc.
4. Compact representation.
5. Typically smaller problem.

![Graph showing TM and TE modes with log10 |λ_n| versus TM/TE modes.](image)

- **Cap.** (capacitive) and **ind.** (inductive) modes.
- Decomposition of **T** and **Z**.
Unification and Properties

\[
Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow T f_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n
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2. Straightforward interpretation.
3. No spurious modes, etc.
4. Compact representation.
5. Typically smaller problem.
6. Only eigenvalue (not generalized eigenvalue) problem.

---

100 1000

\[ \log_{10} |\lambda_n| \]

TM

TE

cap. ind.

doubleprecision

0 20

TM/TE modes

0 20

ind./cap. modes

\( \circ \) decomposition of \( T \)

\( \times \) decomposition of \( Z \)
Properties

\[ Z, U \rightarrow T = -U_1 Z^{-1} U_1^T \rightarrow T f_n = -\frac{1}{1 + j\lambda_n} f_n \rightarrow I_n = -(1 + j\lambda_n) Z^{-1} U^T f_n \]

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2. Straightforward interpretation.
3. No spurious modes, etc.
4. Compact representation.
5. Typically smaller problem.
6. Only eigenvalue (not generalized eigenvalue) problem.
7. Valid for all material formulations.
8. Independent of numerical method (FEM, MoM, FDTD, ...).

Miloslav Čapek, et al.
Unification and Properties

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\[ \log_{10} |\lambda_n| \]

\[ \text{TM/TE modes} \]

\[ \circ \text{decomposition of } T \]

\[ \times \text{decomposition of } Z \]

\[ \text{ind./cap. modes} \]

\[ \text{TM/TE modes} \]

\[ \text{cap. ind.} \]

\[ \text{doubleprecision} \]
Example: Dielectric Cylinder

$\varepsilon_r = 3$, electrical size $kr_\Omega$, $Z$ matrix from PMCHWT transformed into $T$ matrix and decomposed.
Features: Advanced Tracking, FEM Evaluation

Dielectric cylinder, height 4.6 mm, radius 5.25 mm, $\varepsilon_r = 38$, $T$ matrix calculated from $Z$ matrix (PMCHWT), 11 samples, rational fit & interpolation.

Characterstic currents for TE$_{01}$ mode ($f = 4.87$ GHz) and TM$_{01}$ mode ($f = 7.53$ GHz) evaluated with FEM (Comsol Multiphysics) and postprocessed in MATLAB.
Unified Theory ofCharacteristic Modes:
Part I – Fundamentals
Mats Gustafsson, Senior Member, IEEE, Lukas Jelinek, Kurt Schab, Member, IEEE, and Miloslav Capek, Senior Member, IEEE

Abstract—A unification of characteristic mode decomposition for all method of moments formulations of field integral equations describing free-space scattering is derived. The work is based on an algebraic link between impedance and transition matrices, the latter of which was used in early definitions of characteristic modes and is uniquely defined for all scattering scenarios. This also makes it possible to extend the known application domain of characteristic mode decomposition to any other frequency-domain solver capable of generating transition matrices, such as finite difference or finite element methods. The formulation of characteristic modes using a transition matrix allows for the decomposition of induced currents and scattered fields from arbitrarily shaped objects, providing high numerical dynamics and increased stability, removing the issue of spurious modes, and offering good control of convergence. This first part of a two-part paper introduces the entire theory, extensively discusses its properties and offers its basic numerical validation.

Index Terms—Antenna theory, eigenvalues and eigenvectors, computational electromagnetics, characteristic modes, scattering, method of moments, T-matrix method.

1. INTRODUCTION
CHARACTERISTIC modes [1, 2] are established as a useful tool for antenna analysis and synthesis [3, 4]. The number and diversity of applications benefiting from this technique grows rapidly [5] and characteristic mode decomposition is implemented in many contemporary electromagnetic


Unified Theory ofCharacteristic Modes:
Part II – Tracking, Losses, and FEM Evaluation
Mats Gustafsson, Senior Member, IEEE, Lukas Jelinek, Kurt Schab, Member, IEEE, and Miloslav Capek, Senior Member, IEEE

Abstract—This is the second component of a two-part paper dealing with a unification of characteristic mode decomposition. This second part addresses modal tracking and losses and presents several numerical examples for both surface- and volume-based method-of-moment formulations. A new tracking algorithm based on algebraic properties of the transition matrix is developed, achieving excellent precision and requiring a very low number of frequency samples as compared to procedures previously reported in the literature. The transition matrix is further utilized to show that characteristic mode decomposition of lossy objects fails to deliver orthogonal far fields and to demonstrate how characteristic modes can be evaluated using the finite element method.

Index Terms—Antenna theory, eigenvalues and eigenvectors, computational electromagnetics, characteristic modes, scattering, method of moments, T-matrix method.

I. INTRODUCTION
PART I of this two-part paper introduced the theoretical principles of the unified theory of characteristic modes [1], mathematically connecting the impedance [2] and scattering-based [3] approaches to characteristic mode decomposition. This second part develops a procedure for modal tracking based on scattering data interpolation, discusses characteristic modes of lossy systems and their properties, and demonstrates the evaluation of characteristic modes using the complex frequency plane. As a result, the procedure is accurate, computationally inexpensive, simple, and gives excellent results with regards to precision and the low number of frequency samples needed to reconstruct smooth, broadband eigencurves.

Issues resulting from attempts to define characteristic modes for lossy objects, e.g., [11] are examined from the perspective of unified scattering- and impedance-based characteristic mode theory. Thanks to the tight link between the impedance and transition matrices, the physical and algebraic properties of the latter are employed to demonstrate that, for lossy scatterers, it is in general not possible to acquire orthogonal far fields and simultaneously diagonalize the matrix used for the decomposition. This is a fundamental observation which applies to all definitions of characteristic modes for lossy objects, e.g., [11].

The scattering-based decomposition utilizing transition matrix is independent of numerical method used [1]. This feature is demonstrated with the finite element method (FEM) [12, 13] applied to characteristic mode decomposition of a dielectric cylinder. The scattering-based formulation renders characteristic mode theory a general frequency-domain method applicable to various problems, including non-homogeneous dielectrics, in which methods based on differential equations exist.

Characteristic modes...

...are eigenstates of a scatterer transforming incident field to the same field in the far field.
Ongoing Work

Characteristic modes...

...are eigenstates of a scatterer transforming incident field to the same field in the far field.

Scattering dyadics

\[ \int \mathbf{S}(\hat{r}, \hat{r}') \cdot F_n(\hat{r}') \, d\Omega' = \tau_n F_n(\hat{r}) \]

- allows to evaluate characteristic modes, e.g., in FDTD directly,
- to decompose measured data from anechoic chamber!
Conclusion

Characteristic modes

- reveal scattering/radiating mechanisms,
- are useful entire-domain basis,
- excellent for scattering problems,
- applicable to antenna problems,
- are not only “MoM feature”.

Keep in mind that
- original definition is for $T$ matrix,
- original definition has no ambiguities,
- other (useful) decompositions exist.
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- original definition is for $T$ matrix,
- original definition has no ambiguities,
- other (useful) decompositions exist.
Questions?

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December 7, 2021
version 1.1
The presentation is available at capek.elmag.org

Acknowledgment: This work was supported by the Czech Science Foundation (project No. 21-19025M).