Fundamental Bounds for Multi-Port Antennas

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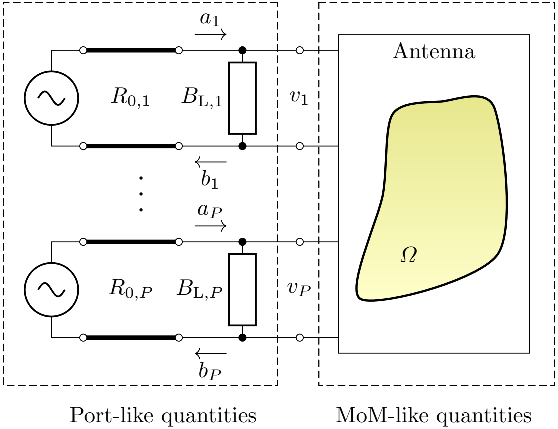
Outline

1. Total Active Reflection Coefficient (TARC)
2. Port-mode Representation
3. QCQP for TARC Minimization
4. Far-field Optimality
5. Concluding Remarks



Multi-port antenna arrangement considered in this talk (MoM part solved full-wave).

Total Active Reflection Coefficient (TARC)

- Useful metric for multiport antennas,
- proposed by Manteghi and Rahmat-Samii[1]:
- problematic to evaluate/measure when ohmic losses are present.

B. TARC

For a desired port excitation, the TARC of the antenna is defined as the square root of the available power generated by all excitations minus radiated power, divided by the available power as follows:

$$\Gamma_a^t = \sqrt{\frac{\text{available power} - \text{radiated power}}{\text{available power}}} = \sqrt{\frac{p_a - p_r}{p_a}}. \tag{7}$$

[1] M. Manteghi and Y. Rahmat-Samii, “Multiport characteristics of a wide-band cavity backed annular patch antenna for multipolarization operations,” *IEEE Trans. Antennas Propag.*, vol. 53, no. 1, pp. 466–474, 2005. DOI: 10.1109/TAP.2004.838794

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TARC as a port quantity

Can we say something about optimality of TARC?

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Total Active Reflection Coefficient – Definition

TARC definition

$$\Gamma^{\mathrm{t}} = \sqrt{1 - \frac{P_{\mathrm{rad}}}{P_{\mathrm{in}}}}$$

Total (input) power

$$P_{\mathrm{in}} = \frac{1}{2}\mathbf{a}^{\mathrm{H}}\mathbf{a}$$

Radiated power (for lossless antenna only!!):

$$P_{\mathrm{rad}} = \frac{1}{2}\left(\mathbf{a}^{\mathrm{H}}\mathbf{a} - \mathbf{b}^{\mathrm{H}}\mathbf{b}\right)$$

Radiated power (always valid)

$$P_{\mathrm{rad}} = \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_{\varOmega}\mathbf{I}$$

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Method of moments

$$\boldsymbol{J}(\boldsymbol{r}) \approx \sum_{n=1}^{N} I_n \boldsymbol{\psi}_n(\boldsymbol{r})$$

$$\mathbf{V} = \mathbf{Z}\mathbf{I}$$

Impedance matrix

$$\mathbf{Z} = \mathbf{R}_{\Omega} + \mathbf{R}_{\rho} + \mathrm{j}\mathbf{X}_{\Omega}$$

Ohmic losses:

$$P_{\mathrm{lost}} = \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_{\rho}\mathbf{I}$$

Port-mode Representation

- We need to express all parameters in terms of port quantities.

Antenna MoM

$$\mathbf{I} = \mathbf{Y}\mathbf{V}$$

Port MoM

$$\mathbf{i} = \mathbf{y}\mathbf{v}$$

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Port-mode excitation

$$\mathbf{V} = \mathbf{C}\mathbf{v}$$

Port current

$$\mathbf{i} = \mathbf{C}^{\mathrm{H}}\mathbf{I}$$

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Port current

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Port admittance matrix

$$\mathbf{y} = \mathbf{C}^{\mathrm{H}}\mathbf{Y}\mathbf{C}$$

Port-mode Quantities

The transformation between current and port quantities generalized

$$\mathbf{I}^{\mathrm{H}}\mathbf{M}\mathbf{I} = \mathbf{v}^{\mathrm{H}}\mathbf{m}\mathbf{v}$$

as

$$\mathbf{m} = \mathbf{C}^{\mathrm{H}}\mathbf{Y}^{\mathrm{H}}\mathbf{M}\mathbf{Y}\mathbf{C}.$$

Port-mode equivalents:

$$P_{\mathrm{rad}} = \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_{\Omega}\mathbf{I} = \frac{1}{2}\mathbf{v}^{\mathrm{H}}\mathbf{g}_{\Omega}\mathbf{v}$$

$$P_{\mathrm{lost}} = \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_{\rho}\mathbf{I} = \frac{1}{2}\mathbf{v}^{\mathrm{H}}\mathbf{g}_{\rho}\mathbf{v}$$

Power waves[2] $\left(\mathbf{\Lambda} = \mathrm{diag}\left(\sqrt{R_{0,p}}\right), \, \mathbf{y}_{\mathrm{L}} = \mathrm{diag}\left(\mathrm{j}B_{\mathrm{L},p}\right)\right)$

$$\mathbf{a} = \frac{1}{2}\left(\mathbf{\Lambda}^{-1}\mathbf{v} + \mathbf{\Lambda}\mathbf{i}\right) = \mathbf{k}_{\mathrm{i}}\mathbf{v}$$

$$\mathbf{b} = \frac{1}{2}\left(\mathbf{\Lambda}^{-1}\mathbf{v} - \mathbf{\Lambda}\mathbf{i}\right) = \mathbf{k}_{\mathrm{r}}\mathbf{v}$$

- Straightforward generalization to, *e.g.*, port stored energy, etc.

[2] D. M. Pozar, *Microwave Enginnering*, 4th. Wiley, 2011

Port-mode Definition of TARC

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Radiation and matching efficiencies:

$$\eta_{\mathrm{rad}} = \frac{P_{\mathrm{rad}}}{P_{\mathrm{rad}} + P_{\mathrm{lost}}} \qquad \eta_{\mathrm{match}} = \frac{P_{\mathrm{rad}} + P_{\mathrm{lost}}}{P_{\mathrm{in}}}$$

Relationship between total efficiency and total active reflection coefficient (TARC):

$$\eta_{\mathrm{tot}} = \eta_{\mathrm{rad}} \eta_{\mathrm{match}} = \frac{\mathbf{v}^{\mathrm{H}} \mathbf{g}_{\Omega} \mathbf{v}}{\mathbf{v}^{\mathrm{H}} \mathbf{k}_{\mathrm{i}}^{\mathrm{H}} \mathbf{k}_{\mathrm{i}} \mathbf{v}} = 1 - \left(\Gamma^{\mathrm{t}}\right)^2$$

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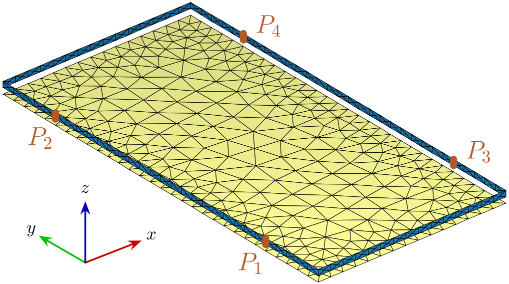
TARC as a port quantity

Maximizing total efficiency η_{tot} means minimizing TARC Γ^{t}.

TARC Minimization

TARC is to be minimized with QCQP[3]:

$$\begin{aligned} &\text{maximize} && \mathbf{v}^{\mathrm{H}} \mathbf{g}_{\Omega} \mathbf{v} \\ &\text{subject to} && \mathbf{a}^{\mathrm{H}} \mathbf{a} = \mathbf{v}^{\mathrm{H}} \mathbf{k}_{\mathrm{i}}^{\mathrm{H}} \mathbf{k}_{\mathrm{i}} \mathbf{v} = 1 \end{aligned}$$



[3] M. Capek, L. Jelinek, and M. Masek, "Finding optimal total active reflection coefficient and realized gain for multi-port lossy antennas," *IEEE Transactions on Antennas and Propagation*, 2021, early access. DOI: 10.1109/TAP.2020.3030941

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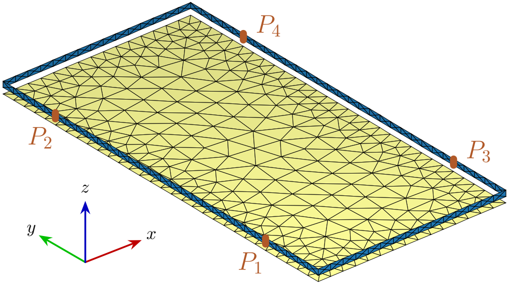

Various levels of complexity:

- optimal excitation of ports,
- optimal placement of ports,
- (optimal) number of ports,
- optimal matching circuitry.

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Simplified Example – Dipole & One Port

One port only $\longrightarrow$ scalar problem $\longrightarrow$ amplitude of feeding voltage is irrelevant:

$$\min_n \left\{ \Gamma_n^{\mathrm{t}} \right\} = \max_n \left\{ \frac{g_{\Omega,n}}{|k_{\mathrm{i},n}|^2} \right\}$$

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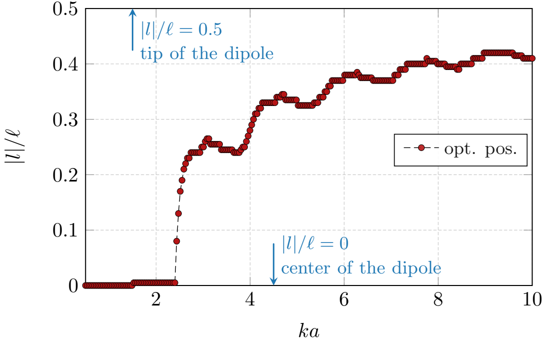
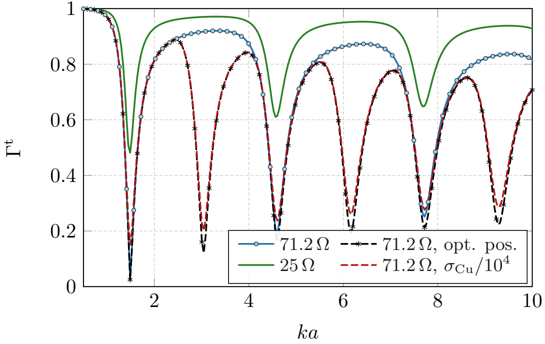




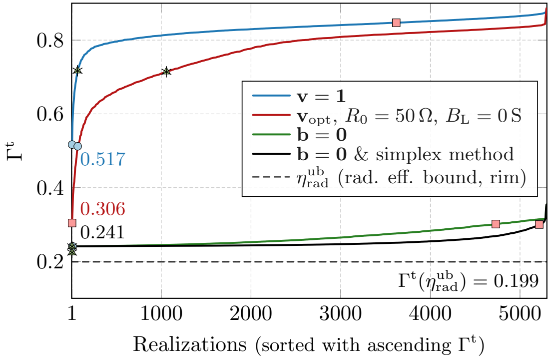
Realistic Example – Metallic Rim

- Optimal placement is a combinatorial problem.
- Optimal voltages found for each combination.

$$\Gamma^{\mathrm{t}} = 0 \quad \Rightarrow \quad \mathbf{b} = \mathbf{0}$$

$$\left(\mathbf{y} + \mathbf{y}_{\mathrm{L}}\right) \mathbf{v} = \left(\mathbf{\Lambda\Lambda}\right)^{-1} \mathbf{v}$$

$$\mathbf{y}\mathbf{v}_i = \left(R_{0\mathrm{L},i}^{-1} - \mathrm{j}B_{\mathrm{L},i}\right) \mathbf{v}_i$$



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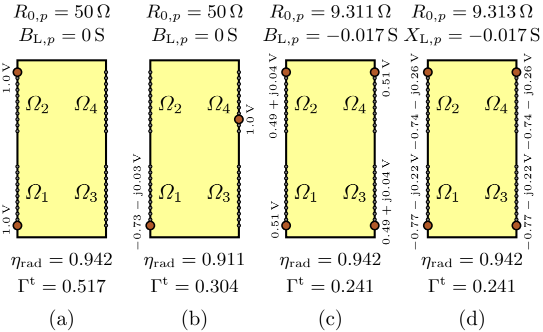

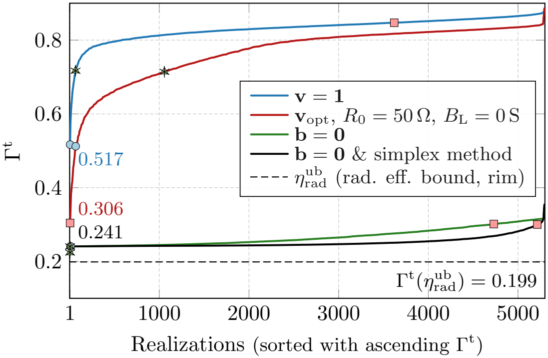

Maximization of Realized Gain

Realized gain for a given excitation:

$$G^{\mathrm{t}} = \left(1 - \left(\Gamma^{\mathrm{t}}\right)^2\right) D = \frac{4\pi}{Z_0} \frac{\mathbf{v}^{\mathrm{H}} \mathbf{f}^{\mathrm{H}} \mathbf{f} \mathbf{v}}{\mathbf{v}^{\mathrm{H}} \mathbf{k}_{\mathrm{i}}^{\mathrm{H}} \mathbf{k}_{\mathrm{i}} \mathbf{v}} = \frac{4\pi}{Z_0} \frac{\left|\mathbf{f}\mathbf{v}\right|^2}{\left|\mathbf{k}_{\mathrm{i}}\mathbf{v}\right|^2}$$

QCQP:

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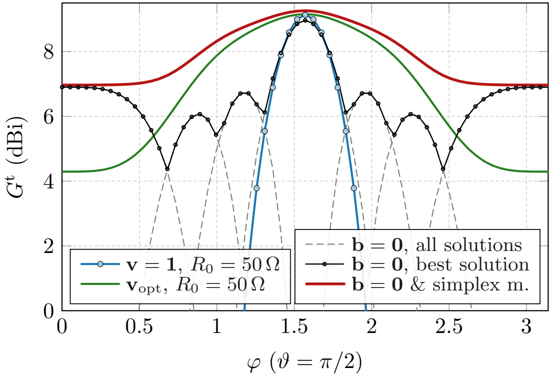


Uniform array of four metallic dipoles, $d = \lambda/2$.

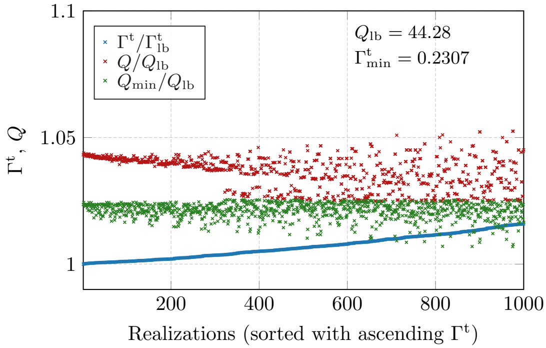
Concluding Remarks

Conclusion

- Changing paradigm from optimal shape to optimal excitation.
- Enormous reduction of the basis.
- The optimal solutions are realizable.
- Optimal placement of feeders possible.

Topics of ongoing research

- Other metrics and their bounds.
- Consideration of realistic (complicated) matching circuits.
- Multi-frequency performance.



Comparison of TARC and Q factor for fed metallic rims. Excitation found via QCQP, placement via combinatorial optimization.

Questions?

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version 1.0
The presentation is available at ▶ capek.elmag.org

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