

# Volumetric Double Negative Metamaterial Composed of Planar Resonators

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**Abstract**— This paper investigates double negative volumetric metamaterials. The first version is a regular 3D periodic structure. This metamaterial does not naturally show an isotropic response. An isotropic response has been obtained for an analog of an amorphous solid material. This metamaterial is composed of planar resonant particles that show negative electric and magnetic polarizabilities. The particles are randomly distributed in space both in their positions and in their orientation. This composite material shows an isotropic response, as it does not prefer any direction of wave propagation. The metamaterial was assembled by inserting planar resonators into polystyrene spheres. The advantage is that the spheres can fill any volume and assure a random distribution of resonant particles both in space and in orientation.

**Index Terms**— Metamaterial, amorphous structure, negative permittivity, negative permeability, isotropic response.

## I. INTRODUCTION

Composite metamaterials as analogs of crystalline solids are composed of periodic systems of resonant elements. By contrast, composite metamaterials in which particular resonant elements are randomly distributed both in space and in orientation are analogs of an amorphous solid material. This system can be used as a metamaterial with an isotropic response. The system composed of broad side coupled split ring resonators (BC-SRR), as presented in [1] – [3], shows negative permeability, and the system composed of planar resonators possessing negative electric polarizability (NEP), similarly as presented in [4], shows negative permittivity. Their isotropic response has already been proven experimentally in [5, 6].

Volumetric isotropic left-handed - double negative (DNG) - metamaterials have been studied by several authors. Zedler et al. [7] presented the rotated transmission-line matrix (TLM) scheme. A metamaterial with an isotropic response composed of transmission lines was designed in [8]. It has been shown that the isotropic response of a 3D system of planar resonant particles located on the faces of cubes is contingent on locating resonators with a proper symmetry [9]. An isotropic response has been reported in the THz and IR bands in [10, 11].

This paper builds on the results of the homogenization procedure proposed in [1, 4], and presents a double negative (DNG) metamaterial composed of BC-SRR and NEP elements [12]. The first DNG metamaterial version has a regular 3D structure with a non-isotropic response. The second version is composed of a mixture of BC-SRRs and NEP elements spread randomly both in space and in orientation. This metamaterial shows an isotropic response and is an analog of the amorphous

solid material.

## II. PLANAR RESONANT ELEMENTS

The BC-SRRs were the same as the BB-SRRs investigated in [1], see Fig. 1a. They were fabricated on a Rogers RT Duroid 5880 substrate 0.127 mm in thickness with permittivity 2.2 and a 0.017 mm copper cladding. Measurements showed that the fabricated BC-SRRs have a normal distribution of resonant frequencies with standard deviation 5 MHz and center value 3.06 GHz. The polarizability of BC-SRRs can be approximated by the Lorentz form

$$\alpha(\omega, \omega_0) = \frac{A\omega^2}{\omega_0^2 - \omega^2 + j\omega\delta} \quad (1)$$

where  $\delta$  represents losses,  $\omega_0$  is resonant frequency, and  $A$  is amplitude. The application of the procedure for determining the free space polarizability presented in [13] defined the particular quantities from (1):  $\omega_0 = 1.922E10 \text{ s}^{-1}$ ,  $A = 5E-8 \text{ m}^3$ ,  $\delta = 0.055 \text{ GHz}$ .

Planar resonant elements [12] were used to produce negative permittivity, see Fig. 1b. These resonators were designed and fabricated using a Rogers RT Duroid 5870 substrate 0.254 mm in thickness with permittivity 2.33. The dimensions of the substrate are 6.7×6.7 mm. The strips and slots are 0.15 mm in width. Measurements of the resonant frequencies gave an average value of 2.965 GHz, with standard deviation  $\sigma = 0.015 \text{ GHz}$ . Measurements of polarizability showed values in Eq. (1) of  $A = 1.28E-18 \text{ m}^2\text{F}$ ,  $\delta = 0.355 \text{ GHz}$ , and  $\omega_0 = 1.92E10 \text{ s}^{-1}$ .



Fig. 1. The fabricated BC-SRR (a), and the fabricated planar NEP resonator [12] (b).

## III. 3D REGULAR SYSTEM

Let us first discuss the response of the regular 3D periodic systems composed of resonant elements from Fig. 1 aligned in the same direction. The experimental realization of this system

is shown in Fig. 2. The resonant elements are inserted into three slices of polystyrene with period 11 mm. The BC-SRRs are located with the substrates parallel to the axis of the waveguide, and the NEPs are located with the substrates perpendicular to the axis of the waveguide. The behavior of the metamaterial was determined by measuring the scattering parameters in the R32 (WG10) waveguide and by simulating their effective permittivity and permeability. Fig. 3 shows the calculated real part of the effective permeability of the metamaterial composed of BC-SRRs, Fig. 1a, and the real part of the effective permittivity of the metamaterial composed of planar NEP resonators from Fig. 1b. Both the permittivity and the permeability are simultaneously negative in the frequency band from 3.09 up to 3.15 GHz. The data plotted in Fig. 3 indicate that the metamaterial composed of the two elements from Fig. 1 a,b will show DNG behavior in the given frequency band. This metamaterial is composed of resonant elements of both types located in the 3D periodic system on an alternating basis.

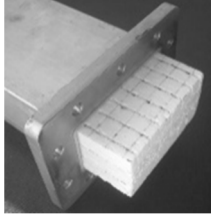


Fig. 2. 3D periodic system of elements aligned in the same direction partly inserted into the waveguide.

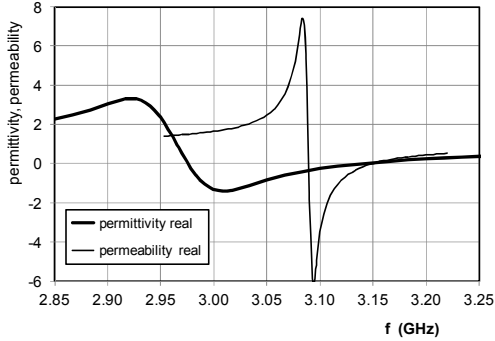


Fig. 3. Calculated real part of the effective permittivity of the regular 3D periodic system of electric dipoles from Fig. 1b aligned in the same direction, and of the real part of the permeability of the system of BC-SRRs located in the regular periodic net aligned all in the same orientation.

The measured scattering parameters of the assembled DNG metamaterial prism with dimensions of  $72 \times 72 \times 33$  mm located in the waveguide are compared in Fig. 4 with the scattering parameters of the 3D periodic systems of the BC-SRRs and of the NEP resonators. The DNG frequency band corresponds to the band in which  $S_{11}$  shows a dip. This band can be defined by condition  $S_{11} < -10$  dB, and spans from 3.1 up to 3.13 GHz. However the band is limited from the low frequency side by the very low value of  $S_{21}$ . The reason lies in the high value of the negative imaginary part of the effective permeability of the

system of BC-SRRs in this frequency band. This metamaterial shows a negative refractive index in the given frequency band of 30 MHz. However, the response is nonisotropic.

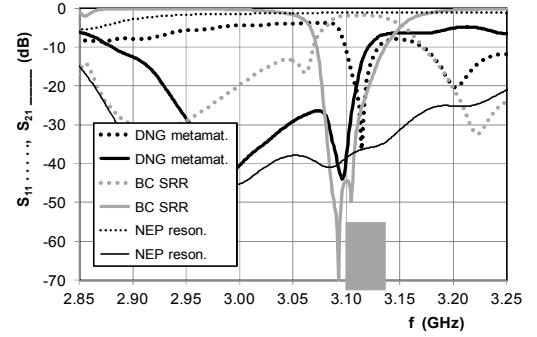


Fig. 4. Measured scattering parameters of regular 3D periodic systems of resonant elements aligned in the same direction: the DNG system, the system composed of BC-SRRs, and the system of NEPs from Fig. 1b. The DNG band is marked by a grey rectangle.

#### IV. AMORPHOUS METAMATERIAL

The amorphous metamaterial was assembled with planar resonators put into polystyrene spheres 10 mm in outer diameter, see Fig. 5. The spheres are cut into two halves and the resonators are glued inside. The diameter of the spheres determines the system period  $a$ . This structure assures random distribution of the resonant elements, both in space and in orientation. The analyzed model is represented by the system of resonant elements located in the 3D net. Their positions are randomly spread around the net nodes in all three directions by  $\pm a/2$  with a uniform distribution of probability, and are randomly oriented in the interval between  $\pm 90$  deg in relation to the  $z$  axis. The measurement setup is shown in Fig. 5. Spheres fill the volume of a cube  $72 \times 72 \times 72$  mm in size that is inserted into waveguide R32 with a modified cross-section 72 mm in height [14]. Tapers between this waveguide and a standard R32 waveguide prevent propagation of the  $TE_{01}$  mode. The amorphous systems composed of BC-SRRs and of NEP resonators [12] were analyzed separately. Fig. 6 shows real parts of the effective permeability and permittivity as functions of frequency. These two systems show the negative real part of the effective permeability (BC-SRRs) and the negative real part of the effective permittivity (NEP from Fig. 1b) simultaneously in the frequency band from 3.124 up to 3.17 GHz. This is the frequency band in which a mixture of resonant elements of both kinds should work as a metamaterial with a negative refractive index. The results of the measurements are shown in Fig. 7. The system composed of both BC-SRRs and NEPs shows  $S_{21}$  much higher than the system of NEPs above 3.15 GHz. In addition,  $S_{11}$  decreases sharply here. This indicates DNG behavior in the frequency band above 3.15 GHz up to about 3.2 GHz, i.e., in the band of about 50 MHz. In Fig. 7, this band is indicated by a grey rectangle.

## V. CONCLUSION

The paper presents two kinds of volumetric metamaterials with simultaneously negative permittivity and negative permeability. These metamaterials are composed of split ring resonators and planar elements with negative electric polarizability. The first version of this metamaterial has a 3D regular structure, where resonant elements with negative electric and magnetic polarizabilities are inserted on an alternating basis in a periodic net. This system shows double negative behavior in a narrow frequency band of 30 MHz. This type of material? is an analog of a crystalline solid material.



Fig. 5. The amorphous metamaterial in the partly disassembled measurement setup.

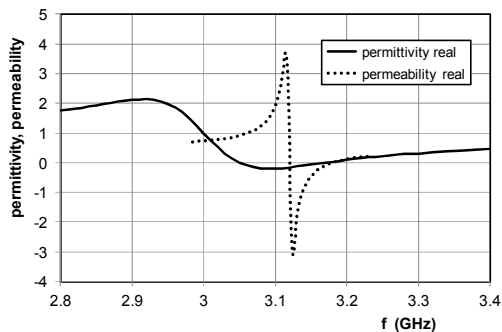


Fig. 6. The calculated real part of the effective permittivity of the amorphous system composed of electric dipoles from Fig. 1b, and the real part of the effective permeability of the amorphous system of BC-SRRs.

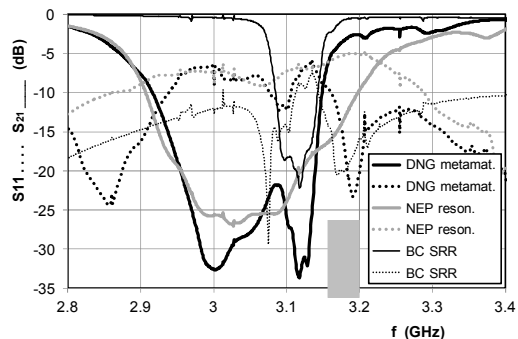


Fig. 7. Measured scattering parameters of the blocks of the amorphous metamaterial located in the R32 waveguide: the DNG system, the system composed of BC-SRRs, and the system composed of NEPs, from Fig. 1b. The DNG band is marked by a grey rectangle.

The designed and fabricated analog of an amorphous solid material is a double negative metamaterial with random distri-

bution of the resonant elements. Resonant elements with negative magnetic and electric polarizability are inserted in polystyrene spheres that are poured into the volume of a cube located in the rectangular waveguide. This structure assures random distribution of the elements both in space and in orientation, and shows an isotropic response. An advantage is that this metamaterial can fill in any volume. There is a narrow DNG band of about 50 MHz above a frequency of 3.15 GHz.

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