Widening the Negative Effective Parameter Frequency Band of Resonant SNG Metamaterials

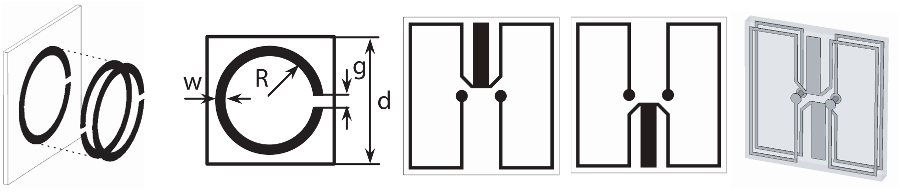
J. Zehentner and J. Machac
Czech Technical University in Prague, Technicka 2, 16627 Prague 6, Czech Republic

Abstract— This paper summarizes some ways of widening the frequency response of epsilon- and munegative metamaterials. The metamaterials are composed of resonant particles and their behavior is theoretically described in terms of coupled resonant circuits theory. The more particles participate in the tested system, the wider the frequency band of the system response. An additional, very important consideration in metamaterial applications is the demand for an isotropic medium. This requirement is therefore also taken into account in our experiments.

1. INTRODUCTION

Volumetric single negative (SNG) metamaterials, i.e., metamaterials with negative permittivity and positive permeability, or vice versa, generally consist of anisotropic resonant particles. There is only a narrow frequency band in which one of the effective parameters of the metamaterials is negative, due to the resonant behavior of the particles, and this seriously limits their technical application. Comparing to this the frequency band of LH planar transmission lines is much wider, even 100% as shown in [1]. These non-resonant structures, however, are not aimed for volumetric applications. This paper presents several ways of widening the frequency band of volumetric SNG metamaterials. This includes widening the medium response resulting from coupling the individual particles as a direct analogy of coupled resonators. The other effect taking place in systems of a large number of particles and leading to band widening is the dispersion of the geometrical and/or material parameters of the particles. In addition, the demand for an isotropic medium is a very important consideration in metamaterial applications.

In experiments to validate the concept of widening the operation band of an SNG metamaterial with negative permeability we used planar broadside coupled split ring resonators (BC-SRR) [2]. The particle consists of two split rings located on the two sides of the substrate, as shown in Fig. 1(a). The metamaterial exhibiting negative permittivity consists of a planar electric dipole loaded by the loop inductor, as shown in Figs. 1(b)–(d). The inductor has two turns utilizing both sides of the substrate in order to shift the medium response of the particle to lower frequencies [3, 4].



(a) (b) (c) (d)

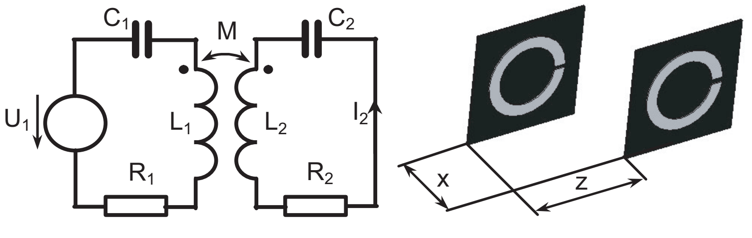
Figure 1: Planar particles used in SNG metamaterials. BC-SRR (a). Planar dipole, top (b) bottom (c) and 3D view (d).

2. A COUPLE OF RESONANT CIRCUITS

One way of extending the frequency response of a resonant circuit utilizes the concept of coupled circuits. The overall transmission of a couple of resonators depends on the coupling coefficient. The transmission admittance of the circuit in Fig. 2(a) is [5]

$$Y_{tr} = \frac{I_2}{U_1} = \frac{jX_v}{R_1 R_2 (1 + jQ_1 F_1)(1 + jQ_2 F_2) + X_v^2}, \tag{1}$$

where $\omega = 2\pi f$, $X_v = \omega M$, $M = k\sqrt{L_1 L_2}$, $\omega_{01,2} = 1/\sqrt{(L_{1,2} + M)C_{1,2}}$, $F_{1,2} = \omega/\omega_{01,2} - \omega_{01,2}/\omega$, $Q_{1,2} = \omega_{01,2}L_{1,2}/R_{1,2}$, k is the coupling coefficient, and f stands for frequency. The frequency band of the response of the circuit from Fig. 2(a) can by widened in two ways. Fig. 3 shows the transmission admittance modulus as a function of frequency (1) in dependence on the coupling coefficient keeping all parameters constant, $R_1 = R_2 = 0.1\,\Omega$, $L_1 = L_2 = 1\,\mathrm{nH}$, $C_1 = C_2 = 2.6\,\mathrm{pF}$ chosen to get the response around 3.1 GHz, similar to the resonant frequency of the investigated particles. The change of the coupling coefficient can be achieved simply by changing the mutual position of particular inductors/particles in space. Fig. 4 documents the influence of capacitance C_1 on the coupled resonant circuits response for $k = 0.004$, and other parameters defined above. This is an analogy to the couple of particles with different parameters, and thus with different resonant frequencies.



(a) (b)

Figure 2: Coupled resonant circuits, equivalent circuit (a), two coupled BCSRRs (b).

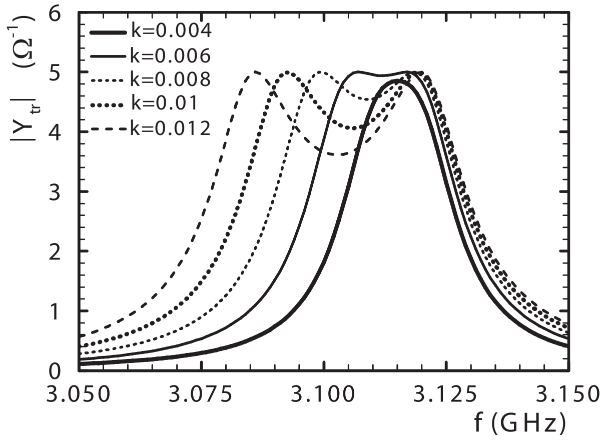


Figure 3: Transmission admittance (1) of the coupled circuits from Fig. 2(a) depending on the coupling coefficient.

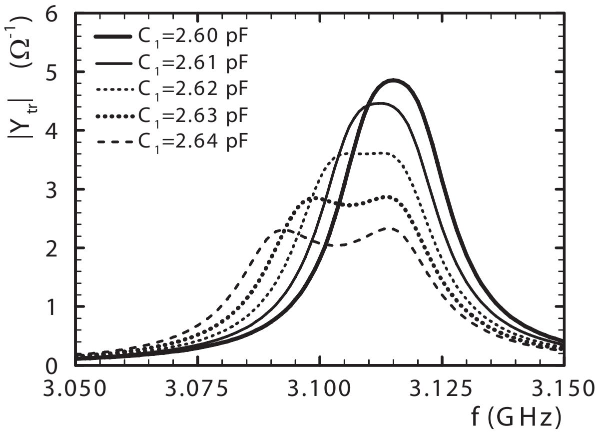
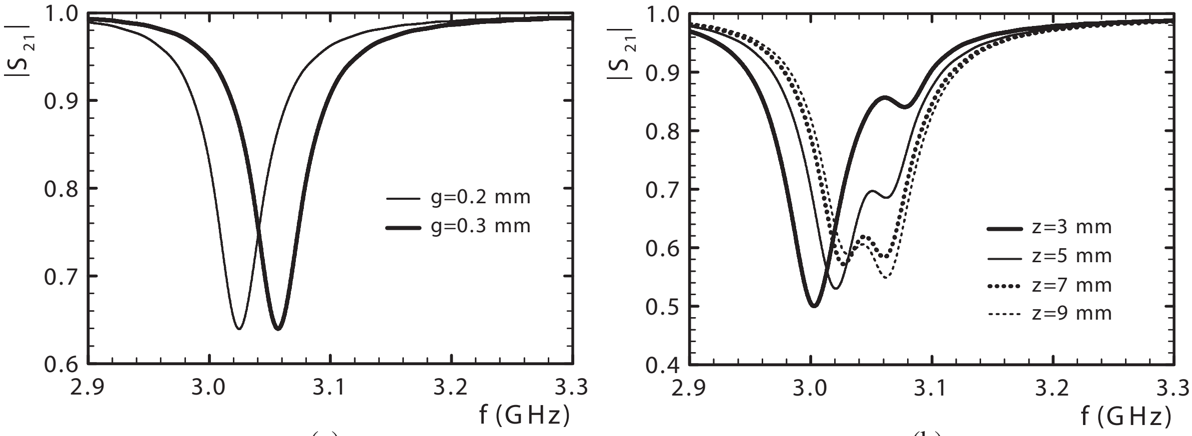


Figure 4: Transmission admittance (1) of the coupled circuit from Fig. 2(a) in dependence on the capacitance C_1.

The equivalent circuit of BC-SRRs is more complex than the simple LC resonant circuit shown in Fig. 2(a). For this reason, the frequency dependence of the transmission coefficient of a pair of BC-SRRs located in a waveguide behaves differently than does a pair of LC resonant circuits. The TEM waveguide with a rectangular cross-section $20 \times 10\,\mathrm{mm}$ was used for the numerical simulation in the CST Microwave Studio. The mutual position of the particles is sketched in Fig. 2(b). The planar particles are parallel to the waveguide side walls and are located at the waveguide center symmetric to the longitudinal waveguide axis. The particles are of the same dimensions: side-width $d = 7\,\mathrm{mm}$ of the squared substrate, 0.127 mm in thickness and permittivity 2.2, ring strip-width $w = 0.7\,\mathrm{mm}$ and inner radius of the ring $R = 1.8\,\mathrm{mm}$. The response of the TEM waveguide with a single BC-SRR for the two different values of the ring split-slot $g = 0.2\,\mathrm{mm}$ and $g = 0.3\,\mathrm{mm}$ calculated by the CST Microwave Studio is shown in Fig. 5(a). The particle is located in the middle of the waveguide parallel to the side walls. The resonant frequency is $f_{r1} = 3.024\,\mathrm{GHz}$ for $g = 0.2\,\mathrm{mm}$ and $f_{r2} = 3.056\,\mathrm{GHz}$ for $g = 0.3\,\mathrm{mm}$. In the case of two particles of the same gap-width located in the TEM waveguide, according to Fig. 2(b), we get different patterns than those shown by the model of the couple of identical LC circuits. The shape of the transmission characteristic of the TEM waveguide with the couple of these particles is not significantly changed by their position, and looks like the transmission characteristic of a single particle. The tighter the coupling, the more the response is detuned towards lower frequencies. This is due to the mutual inductance detuning the circuit. This is shown for the BC-SRRs with the gaps $g_1 = g_2 = 0.2\,\mathrm{mm}$ in Table 1. The transmission characteristics in Fig. 5(b) document what happens when we combine these two effects. The two BC-SRRs are detuned by different gaps 0.2 and 0.3 mm, and their position is varied. For small distances z the mutual coupling of the particles is effective and their frequency response is wider than a single particle response, as shown in Fig. 5(a). At $z \geq 5\,\mathrm{mm}$ the coupling is weak and the final characteristic is the superposition of the particular characteristics, as shown in Fig. 5(a). This, however, results in a sufficiently wider response than a single particle provides.

Table 1: Resonant frequency of the couple of identical BC-SRRs from Fig. 2(b) in dependence on their distance x calculated for $z = 0$.

distance x (mm)	resonant frequency (GHz)
1	2.599
3	2.942
5	3.018
7	3.024



(a) (b)

Figure 5: Calculated transmission of the TEM waveguide with a single BC-SRR of different gap widths (a), two BC-SRRs in dependence on their position defined in Fig. 2(b) for $x = 5\,\mathrm{mm}$ (b).

3. SYSTEM OF PARTICLES

A more effective and lower-cost way of widening the frequency band of the SNG metamaterial response utilizes a system of a large number of particles properly distributed in the hosting material. There are also two mechanisms for widening the metamaterial frequency response that were introduced in the preceding paragraph. In a real system of particles these two effects are combined, and come into effect in the great number of particles periodically or randomly distributed.

The particles in such a system are coupled to each other with various coupling coefficients, and we can observe the same effect as exhibited by a couple of resonant circuits tuned to the same frequency for a certain coupling coefficient determined by the positions of the particles. The particles in the system differ slightly from each other due to the tolerances of the fabrication process in the geometrical dimensions of the conducting layout, and even due to the non-homogeneity of the substrate thickness and its permittivity. Consequently each particle has a different resonant frequency, and the frequency band of the system of particles is wider than that of a single particle. Generally, the more particles are present in the system, the wider the band and the higher the rank of isotropy.

4. EXPERIMENTS AND DISCUSSION

We have verified and evaluated the effects mentioned above for three different composites in the case of both epsilon-negative and mu-negative particles. As the reference, Fig. 6 shows the transmission of one epsilon-negative particle located at the center of the R32 waveguide [4]. The first tested system consisted of a periodic distribution of particles oriented in the same direction. The response of this medium remained anisotropic, as it was for a single particle. Transmission through this system of 147 epsilon negative particles located in an R32 waveguide, see the inset in Fig. 7, is shown in Fig. 7. The frequency band of this medium is considerably wider than that of a single particle, as shown in Fig. 6. At the same time, this system of particles has a more intensive response than that of a single particle.

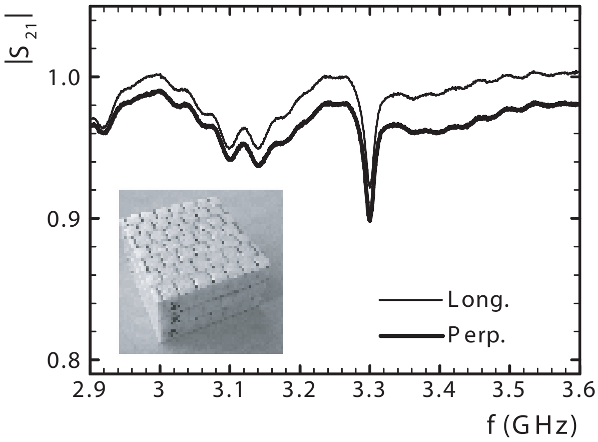


Figure 6: Measured transmission of an R32 waveguide with one epsilon-negative particle (Long. — oriented parallel to the waveguide longitudinal axis, Perp. — located perpendicular to the waveguide axis).

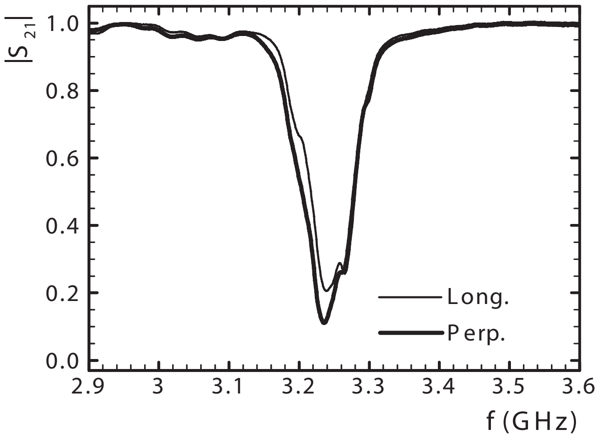


Figure 7: Measured transmission of an R32 waveguide with 147 epsilon-negative particles periodically distributed and aligned in one direction (Long. — located parallel to the waveguide longitudinal axis, Perp. — located perpendicular to the waveguide axis).

As mentioned above, a very important requirement in many metamaterial applications is the isotropy of their response. As stated, the system shown in Fig. 7 behaves anisotropically, i.e., the response depends on the electric field vector orientation. However, the isotropic response was obtained using quasi-randomly [3, 4] or fully randomly [1] distributed particles. Here, in Fig. 8, we present the behavior of a munegative metamaterial by means of the mean value and the dispersion of the transmission through the 64 systems, shown in the inset, and consisting of 243 BC-SRRs located

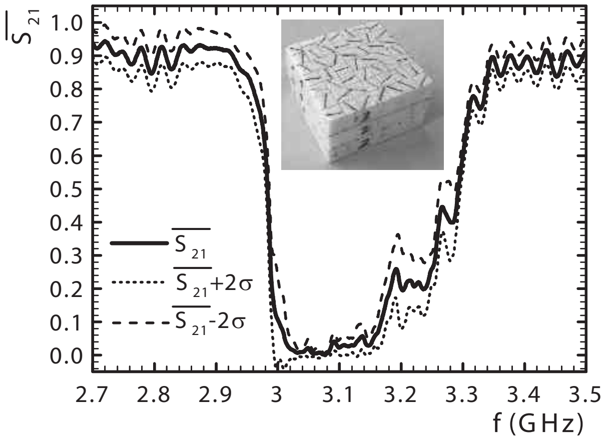


Figure 8: Arithmetic mean value of the transmission through 64 cubic samples in the R32 waveguide with 243 BC-SRRs and its disperse when the resonators are in the nodes of the squared net and are randomly oriented [2].

in an R32 waveguide. These systems were obtained by sequential rotation of particular slices in the sandwich, as shown in Fig. 8. The particles in the slices are located at regular positions, but are randomly oriented. In this way we have obtained a quasi-2D isotropic mu-negative metamaterial. Note the considerably wider frequency band of the metamaterial response compared to the response of a single BC-SRR, which is similar to that shown in Fig. 6 for a single planar dielectric dipole.

The system consisting of 264 BC-SRRs put into plastic shells in the form of a sphere filling the cube with side length 72 mm inserted into the raised R32 waveguide, see Fig. 9, exhibits the transmission plotted in Fig. 10. Now the resonators are fully randomly distributed and oriented, so on an average their mutual coupling is less intensive than in the preceding cases. The response of this system is therefore less intensive and its frequency band is narrower than in the two preceding cases, though still much wider than for a single particle.

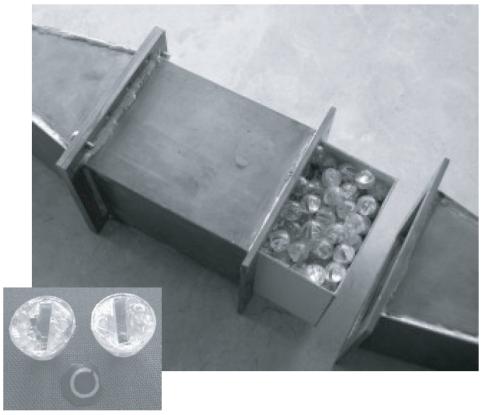


Figure 9: The isotropic mu-negative metamaterial inserted into the partly disassembled measuring setup.

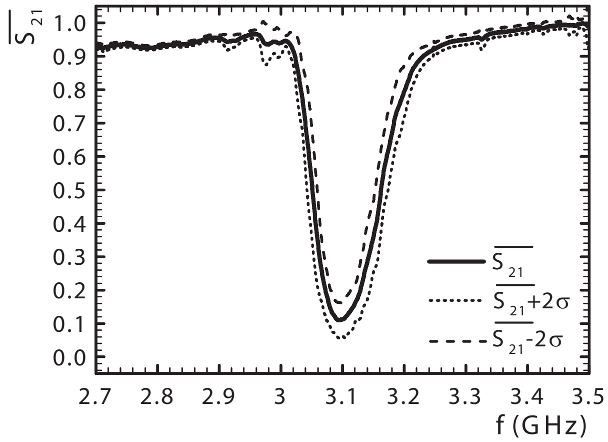


Figure 10: Arithmetic mean value of the transmission through 63 cubic samples in the squared R32 waveguide with 246 BC-SRRs and its disperse when the resonators in the shells are randomly distributed [2].

5. CONCLUSIONS

The paper presents widening of the operation frequency band of SNG metamaterials in terms of the frequency response to irradiation of the composite by the electromagnetic wave. This band is very narrow, due to the resonant characters of the 3D inclusions. According to a simple theory of coupled resonators, one way leads to a change of the coupling coefficient by changing the positions of the particles positions. Within the second way, we changed the resonant frequency of the particles while keeping their position constant. These two approaches were checked and demonstrated by the theory of coupled resonators and by numerical experiments, taking into account magnetic and electric dipoles. The two effects are superimposed in systems of metamaterials composed from a great number of inclusions.

It turned out that the width of the frequency response of the single particle is about 20 MHz, i.e, 0.64% of the resonant frequency. Using the system of 147 epsilon-negative particles periodically distributed and aligned in one direction we got the response-width 110 MHz, i.e., 3.55% relative band, defined at the level -3 dB of the S_{21} modulus. The 2D isotropic system consisting of 147 particles periodically distributed but randomly oriented offered a width of 200 MHz, i.e., 6.45%. The same system consisting of 243 particles offered a width of 330 MHz, i.e., 10.65%. With fully random distribution of particles we got a response-width of 150 MHz, i.e., 4.84% relative band, for a system consisting of 264 BC-SRRs boxed in plastic spherical shells. This is narrower than that for the 2D isotropic system, although we now had more particles. This is due to the less intensive mutual coupling between randomly distributed particles than the coupling between particles distributed periodically.

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