Fourier Transformation and Boundary Conditions in the Special Domain

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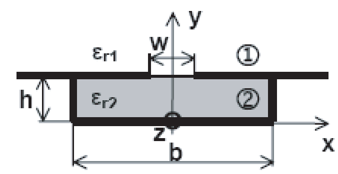

Figure 1: Cross-section of the guide

The method for separating variables in the plane transversal to the field propagation on planar transmission lines is very often not usable. The Fourier transform into the spectral domain provides a solution resulting in the wave equation, where the searched quantity depends only on one variable. Boundary conditions are auxiliary equations enabling the integral and separation constants to be determined. They cannot be formulated easily and clearly, particularly when radiation from the bound region to the boundless region is investigated.

Calculating the propagation constant of a leaky wave propagating in the rectangular waveguide with a longitudinal slot cut in the middle of its wider wall, Fig.1, demonstrates the approach to this problem.

Let $\Psi(x, y)$ represent some strength or potential of the field. Then the Fourier integral fransform is used in the 1st region, and in the x-direction

$$\tilde{\Psi}_1(\xi, y) = \int_{-\infty}^{\infty} \Psi_1(x, y) e^{-j\xi x} dx \tag{1}$$

while in the 2nd region the Fourier finite interval transform is applicable

$$\tilde{\Psi}_2(\xi_n, y) = \frac{2}{b} \int_{-b/2}^{b/2} \Psi_2(x, y) e^{-j\xi_n x} dx \qquad \xi_n = \begin{cases} 2n\pi/b & \text{even modes} \\ (2n-1)\pi/b & \text{odds modes} \end{cases} \tag{2}$$

The distribution of the field in the space domain is obtained by the backward transform. Imposing boundary conditions in the plane y=h transformed to the spectral domain requires, e.g., continuity of the tangential electric field components in the slot $E_{x1}(x, h) = E_{x2}(x, h)$. Of course, $\tilde{E}_{x1}(\xi, h) \neq \tilde{E}_{x2}(\xi_n, h)$ where $\tilde{E}_{x1}(\xi, h), \tilde{E}_{x2}(\xi_n, h)$ are expressed in terms of basis functions. Let us consider $E_{x2}(x, h)$ as the finite function $E_{x2f}(x, h)$ when $|x| \leq b/2$ and $E_{x2f}(x, h)$ equals zero for $|x| > b/2$. After a short manipulation

$$\tilde{E}_{x1}(\xi, h) = \frac{b}{2} \sum_{n=-\infty}^{\infty} \tilde{E}_{x2}(\xi_n, h) \cdot Sa[\frac{b}{2}(\xi - \xi_n)] \tag{3}$$

where the sampling function

$$Sa[\frac{b}{2}(\xi - \xi_n)] = \frac{\sin[\frac{b}{2}(\xi - \xi_n)]}{\frac{b}{2}(\xi - \xi_n)} \tag{4}$$

Parseval's theorem enables current density on the metallization to be eliminated from the boundary conditions. In the end, the standard Galerkin procedure and a complex root search is applied in order to find the propagation constant of the mode.

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