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Loss Compensation in RF Metamaterials by Single Transistor Circuits

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Abstract – **This paper shows that reduction of losses, or even addition of gain, in RF metamaterials can be achieved by a single transistor circuit that exploits its conditional stability. The presented solution offers not only extreme simplicity, but also absolute control over stability and achieved gain values.**

I. Introduction

Losses are certainly one of the biggest practical issues in current metamaterial designs [1, 2]. Unfortunately, losses arise naturally from the assemblies of metamaterial unit cells that are used. With the present state of the art it seems that the only plausible way out of this problem is to use active elements. The first two proposals in this direction [3, 4] come in fact from the very early days of research on metamaterials. They both proposed the use of a Negative Impedance Converter (NIC), which can potentially produce negative capacitances, inductances and resistances and give absolute control over the losses and dispersion of the unit cell. The proposals of [3, 4] were rather academic, since they used an ideal operational amplifier, a component that can (in current technology) hardly be realized for higher frequencies than a few MHz. However, much simpler (two-transistor) designs of NIC are well known in the electrotechnical community [5, 6] which means that this idea could be put into practical use. In fact, designs based on these simplified NICs have already been published [7, 8, 9, 10, 11]. NICs are however not the only components by which negative resistance (and thus loss compensation) can be obtained. Another well-known example is a Gunn diode [12], and there already exist proposals in this direction [13, 14]. However, designs with a Gunn (or tunneling) diode suffer from stability issues. This results from the fact that a Gunn diode offers negative resistance in a very wide frequency range and offers no simple control over it, and thus over the stability of the system. For this reason, the Gunn diode is commonly used in microwave oscillators.

This paper presents another, as yet unexplored, way of achieving negative resistance. It is the use of a conditionally stable transistor. As will be shown, this proposal exhibits the simplicity of the Gunn diode (it uses a single transistor) and at the same time offers absolute control over the stability.

II. Design approach

It is well known [15] that any oscillator can be seen as a connection of an active impedance $Z_\mathrm{A} = R_\mathrm{A} + \mathrm{j}X_\mathrm{A}$ and a loading impedance $Z_\mathrm{L} = R_\mathrm{L} + \mathrm{j}X_\mathrm{L}$. The oscillations will then occur at the frequency where $R_\mathrm{L} + R_\mathrm{A} < 0$ and at the same time $X_\mathrm{L} + X_\mathrm{A} = 0$. Any oscillator thus necessarily contains a negative resistance. As an example, the scheme of a Colpits oscillator [15] is depicted in Fig. 1a. The depicted circuit is intentionally divided into two parts. The left part represents Z_L, while the right part represents Z_A. This division is of course arbitrary, and any separation will lead to a part that has negative resistance. This particular separation is however useful for later considerations. As depicted, Z_A will exhibit a capacitive imaginary part and a negative real part. For an ideal transistor this would be true for any frequency. For a real transistor, the negative real part will be shown only up to some frequency, where the transistor parasitics will enforce negative feedback. The negative real part is bound to the two capacitors of the circuit. If one of them is replaced by an inductance, the negative resistance will disappear completely. Replacing one of the capacitors by a parallel resonant circuit, see Fig. 1b, thus gives the possibility to control the lower limit of the negative resistance band. The positive resistance in the parallel resonance circuit affects the magnitude of the negative resistance of Z_A. The more damped the resonant circuit is, the lower is the magnitude of the negative resistance. In fact, a realistic implementation of Fig. 1b was used in

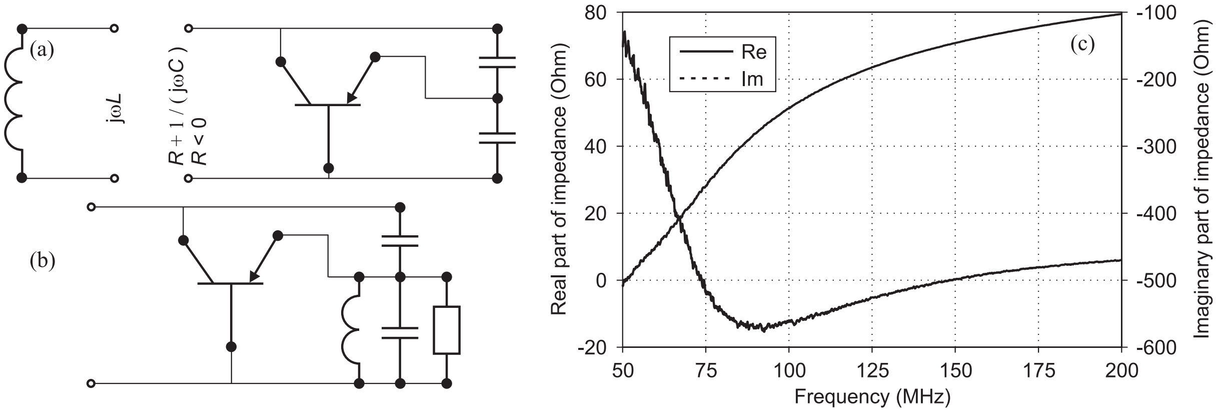

Fig. 1: (a) Scheme of a Colpits oscillator (b) Circuit with band-limited negative resistance (c) Measured input impedance of a realistic implementation of (b)

[16] to create an active magnetic metamaterial particle. In that particular case, the measured input impedance was as depicted in Fig. 1c, where both the capacitive imaginary part and the band limited negative resistance can be clearly seen. Imagine now a cubic array of conductive loops, each loaded by the impedance Z_A of Fig. 1c. The effective permeability of such an array can be estimated as [17]

$$\mu_\mathrm{r} = 1 - \frac{\mu_0 p A_\mathrm{eff}^2 / L}{1 + Z_\mathrm{A} / (\mathrm{j}\omega L) + 2M_\mathrm{a}/L + 4M_\mathrm{c}/L + \mu_0 p A_\mathrm{eff}^2 / (3L)}, \tag{1}$$

where p is the period, L is the inductance of the loop, M_a, M_c are the mutual inductances between the axial and coplanar loops respectively, $A_\mathrm{eff} = N\pi (r_0/p)^2$ is the normalized effective area of the loop, with N and r_0 as the number of turns and the radius of the loop. Depending on the inductance L, the three scenarios depicted in Fig. 2 can arise:

- The net reactance crosses zero below the negative resistance band ($L = 850$ nH). As a result there is $\mathrm{Re}[\mu_\mathrm{r}] > 0,\ \mathrm{Im}[\mu_\mathrm{r}] < 0$ below and in the resonance and $\mathrm{Re}[\mu_\mathrm{r}] < 0,\ \mathrm{Im}[\mu_\mathrm{r}] > 0$ above the resonance.

- The net reactance crosses zero inside the negative resistance band ($L = 300$ nH). As a result there is $\mathrm{Im}[\mu_\mathrm{r}] > 0$ at the resonance and the system is unstable. Each ring becomes an ordinary Colpits oscillator and the medium self-generates a signal.

- The net reactance crosses zero above the negative resistance band ($L = 155$ nH). As a result there is $\mathrm{Re}[\mu_\mathrm{r}] > 0,\ \mathrm{Im}[\mu_\mathrm{r}] > 0$ below the resonance and $\mathrm{Re}[\mu_\mathrm{r}] < 0,\ \mathrm{Im}[\mu_\mathrm{r}] < 0$ in and above the resonance.

Clearly, only the 1st case and 3rd case are of interest for metamaterial design. Of course all the above considerations are not limited to magnetic properties only. A simple change from a Colpits oscillator to a Hartley oscillator (exchanging the two capacitors in Fig. 1a for two inductors) [15] will lead to an active impedance with a negative real part and an inductive imaginary part being an immediate candidate for a load of capacitive scatterers and thus for active permittivity materials.

III. Conclusion

It has been shown that NICs and Gunn diodes are not the only candidates for active metamaterial designs. In fact there exists a much more appealing alternative (in terms of simplicity, achieved bandwidth and stability): a conditionally stable transistor. The proposal has been presented on a particular example of an active magnetic metamaterial made of loaded conducting rings, but it is not limited to this example and has quite general validity.

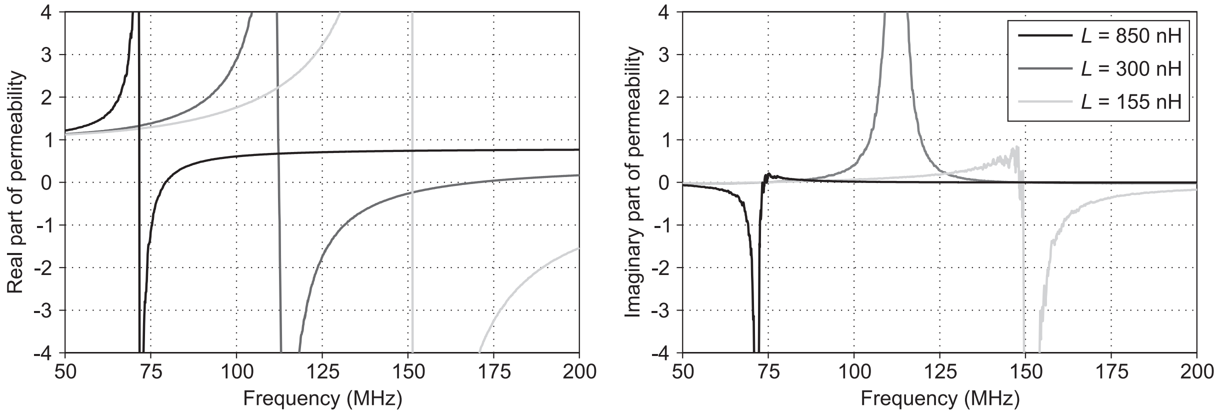

Fig. 2: Calculated permeability of a cubic lattice of active rings with $N = 4$, $p = 30$ mm, $r_0/p = 0.42$, $M_\mathrm{a} = 19$ nH, $M_\mathrm{c} = -25$ nH and three different inductances L. The values correspond to the particular design of [16] with impedance Z_A according to Fig. 1c. Note that the curve for $L = 300$ nH is just a mathematical result of Eq. 1 and does not exist in reality, since the system is unstable for this L.

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