

Metamaterial Made of BC-SRRs with Randomly Dispersed Resonance Frequencies

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Abstract — This paper reports on an investigation of a metamaterial designed as a 3D system of broad side coupled split ring resonators. Imperfections in the fabrication process cause dispersion of the geometrical and material parameters of the resonators, and thus dispersion of their resonance frequencies. Homogenization of a 3D cubic lattice of these imperfect resonators reveals that the resulting resonance frequency band of the effective permeability is significantly widened, but at the cost of reducing its magnitude or even completely disappearing the negative values of its real part. An experiment performed in the rectangular waveguide verifies the theoretical predictions.

Index Terms — Metamaterial, homogenization, effective permeability, split ring resonator, randomness.

I. INTRODUCTION

At microwave frequencies, volumetric metamaterials are mostly designed as 3D periodic systems of resonance particles showing negative electric and/or magnetic polarizabilities. However, their resonance nature makes the response of the particles rather sensitive to fabrication precision, which mostly depends on the precision of the printed circuit board process and the way in which 3D lattice of the resonators is assembled. Variations in the geometry and in the material parameters of particular resonators can be taken into account by assuming resonators with dispersed resonance frequencies [1]. However, an imperfectly assembled 3D periodic structure is a structure with randomly distributed and/or randomly oriented resonators. Both of these effects widen the frequency band of the metamaterial response and reduce the magnitudes of its effective parameters. The frequency band of negative effective permittivity and/or permeability can even disappear. At the same time, random distribution of resonance particles can be intentionally used to design a metamaterial with an isotropic response, as has been shown experimentally in [2].

Less attention has been paid to investigations of metamaterials with randomly distributed resonant particles or with randomly dispersed resonant frequencies. A metamaterial composed of a basically periodic system of cut-wire pairs was investigated in [3]. Systems of randomly oriented chiral particles were studied in [4]. The transmission properties of waveguides with a statistical disorder in their periodicity were studied in [5]. A fully experimental investigation of randomness in metamaterials was performed in [6-10]. Another group of works has investigated periodic structures composed of elements differing in their parameters, e.g. in their geometrical dimensions. Gollub et al. [9] used the

Bruggeman mixing model, and showed widening and finally disappearance of the band of negative permeability by increasingly modifying the dimensions of a split ring resonator (SRR) in a periodic system. Fujii [10] made an experimental study of the focusing property of a 2D LH superlens, introducing fluctuation into the capacitance values, and examining the distortion in the focused electromagnetic images. Papers [1, 11] investigated the 3D periodic system of SRRs. Fluctuations of SRR parameters were represented by fluctuations of their resonance frequencies. The authors showed that microscopic averaging can be performed prior to statistical averaging only under the assumption that the resonance frequency is a slowly varying random function of the coordinates. An increasing level of uncertainty dramatically changes the permeability dispersion, and the band of negative permeability finally disappears.

This work studies the behavior of a homogeneous 3D periodic system of broad side coupled SRRs (BC-SRR) aligned in one direction. The dispersion of their parameters is represented by the corresponding dispersion of the resonance frequencies. BC-SRRs are treated as point magnetic dipoles with their polarizability determined by a simple analytical formula [12]. Homogenization of this periodic system is performed using the classical Lorentz homogenization procedure [13]. Particular resonators are taken with randomly distributed resonance frequencies. The results of the analysis are verified by measuring the transmission of the system of BC-SRRs inserted in a rectangular waveguide. This analysis forms the first step of an investigation of an amorphous structure in which particular resonance elements are randomly located. This amorphous material is a good candidate for obtaining a practical design for a metamaterial with an isotropic response.

II. HOMOGENIZATION PROCEDURE

Let us assume a system composed of BC-SRRs located in a 3D cubic lattice of period a with their axes aligned in the z direction. Period a is assumed to be much smaller than the free space wavelength. The resonators are represented by point magnetic dipoles with polarizability [12]

$$\alpha = \frac{A\omega^2}{\omega_0^2 - \omega^2 + j\omega\delta} = f(\omega, \omega_0), \quad (1)$$

where ω and ω_0 are angular and resonance frequencies, respectively, and A is positive amplitude. The excitation and also the total averaged magnetic field $\hat{\mathbf{H}} = \hat{H} \mathbf{z}_o$ is assumed to be aligned with the dipole orientation. Averaging is taken over the cell volume $V = a^3$, $\hat{H} = \langle H \rangle = \frac{1}{V} \iiint_V H dV$. The magnetic field exciting the reference magnetic dipole located at the origin of the coordinate system is according to [13]

$$H_z^{(ind)} = \hat{H}_z + \sum_{i \neq 0} (H_{iz} - \hat{H}_{iz}) - \hat{H}_{0z}. \quad (2)$$

Field H_{iz} is the field at the position of the reference dipole excited by a point magnetic dipole located at position i , and can be expressed by [14]

$$H_{iz} = m_i F_i^{(zz)}. \quad (3)$$

In (2), these fields are summed over all magnetic dipoles, with the exception of $i = 0$, as the dipole cannot excite itself. The average value of the field excited by the reference magnetic dipole is [14]

$$\hat{H}_{0z} = \frac{2}{3a^3} m_0. \quad (4)$$

Magnetic field (2) defines the magnetic moment of the reference dipole. It is assumed in the following text that all magnetic dipoles are excited by the same field, so there is

$$m_i = \alpha_i H_z^{(ind)} = f_i(\omega, \omega_{oi}) H_z^{(ind)}. \quad (5)$$

Inserting now (3) and (4) into (2) using (5) one can get the magnetizing field $H_z^{(ind)}$ and finally m , the magnetic moment of the reference dipole. The effective permeability is determined from the relation

$$B = \mu_o (\hat{H} + M) = \mu_o \left(\hat{H} + \frac{m_0}{a^3} \right) \quad (6)$$

$$\mu_{zz}(\omega, \omega_o) = 1 + \frac{f_0(\omega, \omega_o)}{a^3 \left(1 + \frac{2}{3a^3} f_0(\omega, \omega_o) - \sum_{i \neq 0} f_i(\omega, \omega_{oi}) \left(F_i^{(zz)} - \langle F_i^{(zz)} \rangle \right) \right)}, \quad (7)$$

f_0 stands for the function in (1), defined for the reference magnetic dipole, whereas f_i is defined for the i^{th} dipole. Due to the symmetry of the system, all other components of the permeability tensor are equal to zero. Finally the effective permeability is determined by integrating (7) multiplied by the distribution function of the resonance frequencies. In the summations of (2), the resonance frequencies are randomly chosen around the central value, independently for each magnetic dipole, and (2) is averaged over a chosen number of realizations. This replaces N dimensional integration. The resulting process has fast convergence. In this way, the randomness of resonance particles is built into the homogenization process, as stated in [11].

III. ANALYSIS AND EXPERIMENT

The analyzed periodic system consists of BC-SRRs, which were investigated by Jelinek et al. in [2], see Fig. 1. The resonators are fabricated on a Rogers RT Duroid 5880 substrate 0.127 mm in thickness with relative permittivity 2.2 and 0.017 mm copper cladding. The dimensions according to Fig. 1 are: $R = 1.8$ mm, $w = 0.7$ mm, $g = 0.3$ mm, $d = 7$ mm. At a low frequency limit, the polarizability of this planar resonator can be described by analytical expression (1). Application of the procedure determining the free space polarizability described in [15] gives the particular parameters: $\omega_0 = 1.9515 \cdot 10^{10} \text{ s}^{-1}$, $A = 5 \cdot 10^8 \text{ m}^3$, $\delta = 5.5 \cdot 10^7 \text{ s}^{-1}$. Resonance frequencies of 149 BC-SRRs were measured in order to determine their distribution caused by imperfect technology. This function has been fitted by the Gaussian distribution with the mean value equal to 3.106 GHz, and with standard deviation equal to 0.0252 GHz, see the plot in Fig. 1.

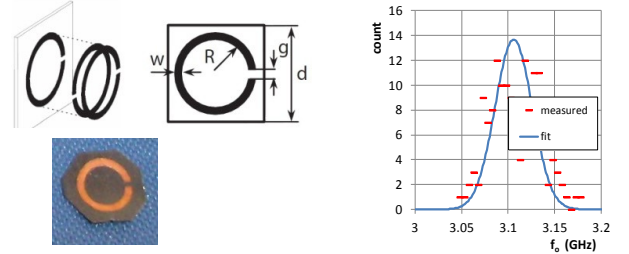


Fig. 1. A planar broad-side coupled split ring resonator, and the distribution of the measured resonance frequencies with their fit.

The procedure presented in the previous paragraph is applied using the following details. The summation in (2) is taken over a sphere of radius $\rho = 9a$ with its center at the position of the reference magnetic dipole. The resonance frequencies of particular BC-SRRs are randomly selected with the above given normal distribution. The final sums are averaged over 80 statistical realizations. The radius ρ and the number of statistical realizations have been set large enough to obtain convergent results.

The behavior of the metamaterial composed of the BC-SRRs defined above with dispersed resonance frequencies is documented in Fig. 2. Increasing dispersion of the resonance frequencies of the particles reduces the absolute effective permeability value and widens the frequency band around the resonance. For standard deviation greater than or equal to 0.06 GHz, the band of negative effective permeability disappears. The negative maximum of the imaginary part of the effective permeability representing the losses increases with decreasing standard deviation. However, its integral taken over the whole frequency band is constant, as it corresponds to the total loss.

The scattering parameters of a metamaterial prism of dimensions $70 \times 70 \times 33$ mm were measured in a rectangular waveguide with metallic walls R32 in order to verify the theory presented here. This prism is assembled by inserting resonance BC SRRs into three slices of polystyrene 11 mm in thickness, see the inset of Fig. 3. The results of this

measurement were compared with the scattering parameters calculated by the CST Microwave Studio, applying the complex permeability determined by the homogenization process presented above.

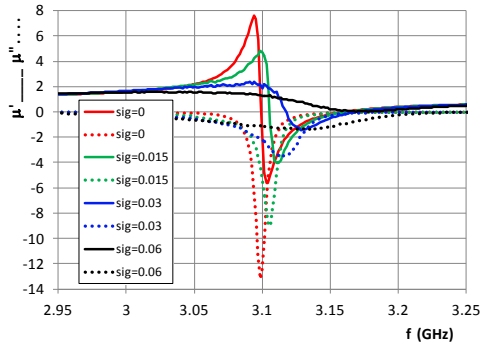


Fig. 2. Calculated effective permeability of a metamaterial with period $a = 11$ mm as a function of the standard deviation denoted by the legend as sig .

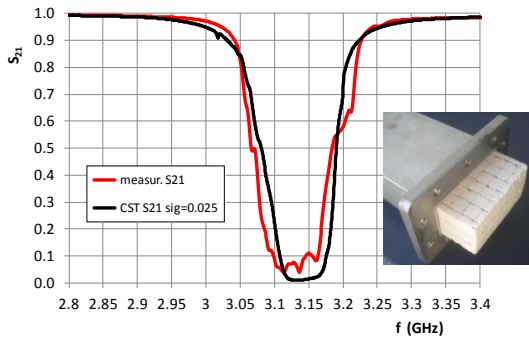


Fig. 3. Measured and calculated scattering parameters of a metamaterial prism with period $a = 11$ mm. The inset shows the measured sample.

IV. CONCLUSION

Volumetric metamaterials are mostly designed as periodic 3D systems of resonance particles – here, planar BC SRRs. The particles are always fabricated with some dispersion of their dimensions and material parameters. This dispersion is represented by the corresponding dispersion of resonance frequencies. Measurements of a selected number of BC-SRRs showed that this dispersion has a normal distribution. The randomness was incorporated into the process of metamaterial homogenization. In this process, BC-SRRs were represented by point magnetic dipoles. The complex effective permeability of the metamaterial was calculated. The simulation process presented here works effectively with fast convergence.

The random distribution of resonance frequencies of particular magnetic dipoles – BC-SRRs – significantly widens the resonance frequency band of the effective permeability of the metamaterial. At the same time, it reduces the magnitude of the effective permeability, and the frequency band of its negative value is narrowed or this band even disappears at higher standard deviation values of the resonance frequencies. This effect has been verified experimentally by measuring the scattering parameters of the metamaterial prism in the

rectangular waveguide and comparing these results with the parameters calculated by the CST Microwave Studio using the calculated complex permeability dispersion.

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