Building Elements of a Volumetric Metamaterial

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***Abstract* — This paper presents the results of an investigation into a combination of electric and magnetic planar resonators in order to design the building element of a volumetric metamaterial showing simultaneously negative electric polarizability and magnetic polarizability under irradiation by an electromagnetic wave. The magnetic element is a split ring resonator, and the electric element is an electric dipole terminated by an inductor. The response of the single resonant particle is strongly anisotropic. Proper spatial arrangement of these particles can make the response isotropic. The possibility of obtaining a non-bianisotropic response is taken into account.**

***Index Terms* — metamaterial, resonant particle.**

I. INTRODUCTION

The elementary cell of a volumetric metamaterial consists either of a single specifically shaped element, or of a suitably spatially arranged set of particles [1,2]. For most known applications, an isotropic metamaterial is required. Isotropy can be achieved by periodic arrangement of the inclusions satisfying symmetry of the selected crystallographic group [3-5], or by the use of randomly oriented inclusions in the volume of the host [6]. However, random arrangements are not easily reproducible and do not show a well-defined electromagnetic response, mainly if the size of the inclusions is not sufficiently small compared to the wavelength. Periodic systems are thus usually the preferred option.

Apart from isotropy, another frequent requirement is that the metamaterial will not exhibit magneto-electric coupling, i.e., a non-bianisotropic material is required. In this case, inversion symmetry of the inclusions is required [7].

There are basically two candidates for the metamaterial defined above in the present state-of-the-art. The first candidate is a combination of properly shaped split ring resonators (SRR) [3] in combination with a 3D wire medium [8]. The second candidate is an arrangement of dielectric or magneto-dielectric spheres [9, 10]. Unfortunately, neither of those two options is easy to make in practice. The problem with the first option is the great complexity of the 3D wire mesh. The second option requires either magneto-dielectric materials or precisely-shaped dielectric objects made from high permittivity materials. In both cases the technological demands are high.

The objective of this paper is to investigate and design a particle that will simultaneously show negative electric and magnetic polarizabilities – double negative (DNG) behavior. In addition, we will require that the particle can be manufactured by standard techniques for planar circuits. This resonant particle is composed as a combination of an SRR and an electric dipole (ED) terminated by an inductor. Finally, the version of the resonant particle that will exhibit inversion symmetry to assure a non-bianisotropic response will be presented.

II. INVESTIGATION AND DESIGN OF RESONANT PARTICLES

The aim of our work was to design a resonant particle showing simultaneously negative electric and magnetic polarizabilities induced by an exciting electromagnetic wave, i.e., a DNG response. SRR [3] and ED [4] are suitable building blocks for this element. The layouts of these planar resonators are drawn in Fig. 1. A ceramic substrate with ε_r = 20 was used due to very low losses, as its loss factor is 10^{-4}. This substrate has the shape of a disk 7 mm in radius and 2 mm in thickness.

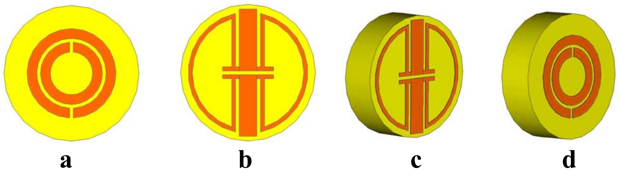


Fig. 1 Layouts of SRR (a), and of ED (b), the front (c) and the rear (d) view of the particle.

The resonant particles were analyzed by the CST Microwave Studio when located in the center of an ideal TEM wave waveguide with perfect electric walls on top and bottom, and perfect magnetic walls on its sides. The waveguide was 20 mm in length, 10 mm in height, and 10 mm in width and was filled with air. The scattering parameters result from this analysis. A very simple criterion was applied for predicting the character of the resonant particle behavior, i.e., for determining the effective refractive index of the cell with the particle [11]. This assumes geometrically symmetric cells, so that S_{11} = S_{22} and S_{21} = S_{12}, with low losses assuring high transmission and low reflection. Under these circumstances, the effective refractive index of the cell n is

$$n = \frac{-\arg S_{21}}{k_0 d} \quad , \qquad (1)$$

where, k_0 is free space propagation constant and d is the cell length, represented by the distance of the reference planes. Consequently the cell behavior is fully determined by the phase of the transmission coefficient.

The main problem in resonant particle design is due to mutual coupling of a particular ED and SRR. This coupling detunes the final particle composed of SRR and ED and causes the existence of two separate resonances. The two building blocks - SRR and ED - were first studied separately and tuned to the same resonant frequency 2.9 GHz when located on a disk substrate 0.4 mm in thickness. These two resonators were located at the TEM waveguide in parallel, back to back. Here the coupling is negligible when their mutual distance is greater than approx. 1mm and the resonant frequency remains 2.9 GHz. Reducing the substrate distance, e.g., to 0.5 mm, we get magnetic and electric resonant frequencies of 2.84 GHz and 2.915 GHz, respectively. This arrangement of course does not meet the objective of the design, as we do not have a single solid particle. The planar resonators have to be located on a single substrate, see Figs. 1c,d. In this case, the coupling is unfortunately stronger. To reduce the influence of this effect, the substrate thickness was increased to 2 mm in the final design.

The final DNG resonant particle was composed of ED with the following parameters: dimensions of dipole arms 2.95 mm in length, 0.9 mm in width, radius of inductive strip 3.1 mm and 0.2mm in width, width of slots 0.2 mm, length of horizontal stubs 2.6 mm. SRR has the following geometrical dimensions: outer radius 2.3 mm, strip width 0.5 mm, slot width 0.2 mm. Fig. 2 shows the scattering parameters of the TEM waveguide with the particle located in its center, calculated by the CST Microwave Studio.

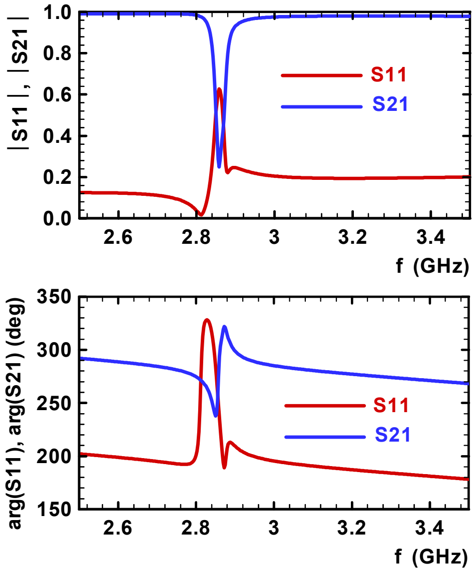


Fig. 2 Calculated transmission characteristics of the DNG resonant particle defined in the text.

The electric α_{yy}^{ee} and magnetic α_{xx}^{mm} polarizabilities of the particle were determined from the scattering parameters plotted in Fig. 2, according to the procedure presented in [12]. Reduced electric polarizability $\alpha_{yy}^{ee}/\varepsilon_0$ is plotted in the following plots (without notification) in order to be directly comparable with the magnetic polarizability value, ε_0 being vacuum permittivity. These values are plotted in Fig. 3 in dependence on frequency. In the frequency band from 2.87 up to 2.89 GHz the particle shows DNG behavior. The effective permittivity ε_r and permeability μ_r, and therefore the effective refractive index, of the waveguide cell containing the resonant particle were calculated from the polarizabilities using the simple Lorentz homogenization technique [13]

$$\varepsilon_r = 1 + \frac{\alpha_{yy}^{ee}}{V\varepsilon_0 - \alpha_{yy}^{ee}/3} \;, \tag{2}$$

$$\mu_r = 1 + \frac{\alpha_{xx}^{mm}}{V - \alpha_{xx}^{mm}/3} . \tag{3}$$

The values of these effective parameters depend on the cell volume V, over which they are averaged. The values plotted in Fig. 4 were calculated for the volume of a cube 7 mm in edge length.

A certain drawback of the designed resonant particle lies in the high sensitivity of its response to some layout geometrical dimensions.

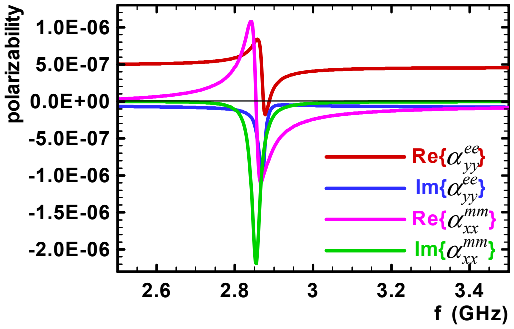


Fig. 3 Electric and magnetic polarizabilities of the resonant particle calculated from the scattering parameters in Fig. 2.

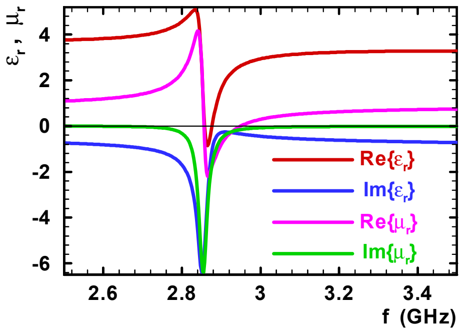


Fig. 4 Effective permittivity and permeability of the resonant particle calculated from the polarizabilities in Fig. 3 by (2) and (3).

III. EXPERIMENT

The designed resonant planar particles were fabricated using a standard lithography technique using silver layouts on disk-shaped E-20 ceramic substrates [14]. The parameters of this material are defined above.

The behavior of the resonant particles located in the TEM waveguide was tested by measuring the scattering parameters. The waveguide cross section was 20 times 8 mm, and the distance of the reference planes was 7 mm.

Some of particles were detuned due to the fabrication tolerances, so that there were two separate resonances in the measured characteristics, one electric and one magnetic. This is documented by the dependence of the polarizabilities on frequency for a selected particle specimen in Fig. 5. Magnetic resonance occurs at 2.93 GHz, while electric resonance occurs at 2.72 GHz. The polarizabilities plotted in Fig. 5 were calculated from the measured S parameters, as stated above.

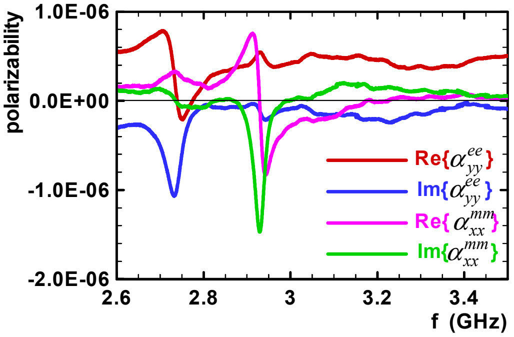


Fig. 5 Electric and magnetic polarizabilities of the resonant particle not properly tuned, calculated from measured scattering parameters.

Changing the length of the horizontal stubs in ED makes it possible to tune the electric resonance and to force the particle to resonate at a single frequency, and therefore to posses DNG behavior. This can be done on a very fine scale, even with fabricated particles, by milling away part of the horizontal stub metal and moving the electric resonance up, as the capacitance can be only reduced in this way.

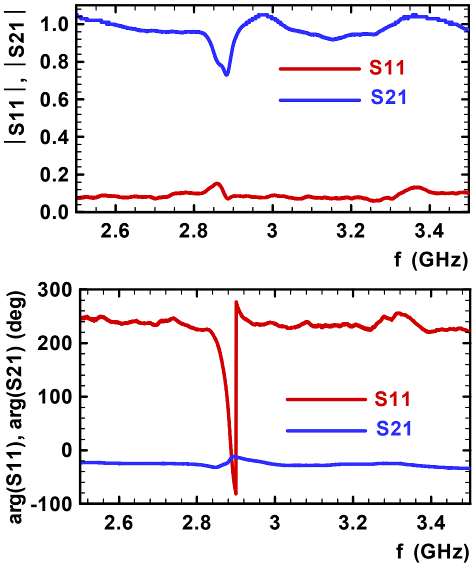


Fig. 6 Measured transmission characteristics of the DNG resonant particle tuned to 2.9 GHz.

The measured transmission characteristics of a selected particle tuned to 2.9 GHz are plotted in Fig. 6. The real parts of the polarizabilities corresponding to the S-parameters shown in Fig. 6 are plotted in Fig. 7. This resonant particle shows a DNG response in the band between 2.87 and 2.95 GHz.

The real part of the effective permittivity calculated by (2) and (3) over a cube with edges 7 mm in length using the polarizabilities in Fig. 7 reaches a minimum value of -0.99 at 2.89 GHz and a minimum effective permeability value of -1.49 at 2.9 GHz (not shown in the plots). The values calculated over the volume of the whole waveguide cell with cross section 20x8 mm and 7 mm in length are plotted in Fig 8. This plot shows a very good match of the refractive indices calculated using (1) marked here as $n(S_{21})$ and using real parts of permittivity and permeability in Fig. 8 getting $n = \sqrt{\mathrm{Re}(\varepsilon_r)\mathrm{Re}(\mu_r)}$. As stated above, these values are positive due to higher cell volume.

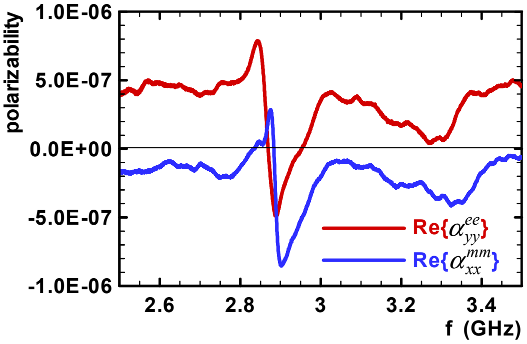


Fig. 7 Real parts of the electric and magnetic polarizabilities of the resonant particle showing DNG behavior. Imaginary parts are not shown, in order to avoid having an excessively dense plot.

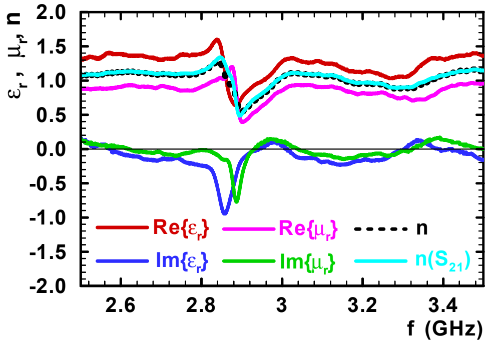
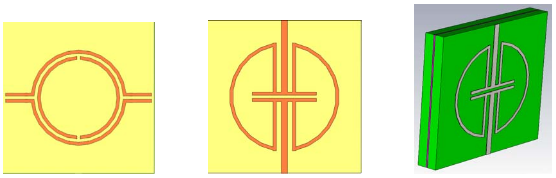


Fig. 8 Effective permittivity, permeability, and refractive index of the particle from Figs. 6, 7.

IV. NON-BIANISOTROPIC PARTICLE

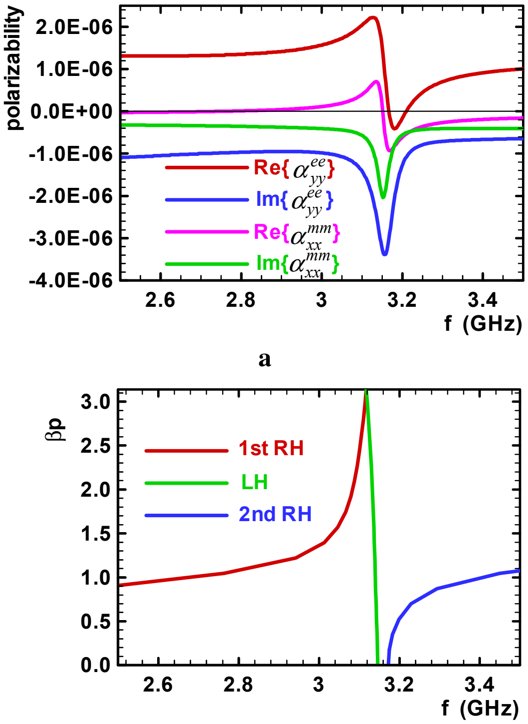
The particle has to possess inversion symmetry [7], in order not to exhibit magneto-electric coupling. In this case a proper spatial arrangement of SRR and ED must be used. The modified symmetric SRR layout is now used, as shown in Fig. 9a. This SRR is sandwiched between two dielectric chips 0.625 mm in thickness and 9.5 mm in size, and with permittivity equal to 10. The two EDs, see Fig. 9b, are located on the outside surfaces. A view of this element is shown in

Fig. 9c. The tuning process for the planar resonators was performed, setting their capacitances by changing the lengths of the horizontal stubs. The polarizabilities and corresponding dispersion characteristics of the resonator are plotted in Fig. 10. The polarizabilities were determined by the technique presented in [12], using the scattering parameters calculated by the CST Microwave Studio. The LH mode propagates in a narrow frequency band shown by a green line in the dispersion characteristic, see Fig. 10b. This corresponds to the frequency band of simultaneously negative polarizabilities. The dispersion plot in Fig. 10b belongs to a wave propagating through the TEM waveguide cell containing the particle, with the electric field parallel to the substrate and the magnetic field perpendicular to the substrate.



a **b** **c**

Fig. 9 Layouts of SRR (a), ED (b), and particle composition (c).



a

b

Fig. 10 Electric and magnetic polarizabilities of the resonant particle defined in the text (a), and the dispersion characteristic (b).

V. CONCLUSIONS

This paper presents a resonant particle with double negative behavior suitable for the design of volumetric metamaterials. This particle is composed of an SRR and a dielectric dipole located on a single disk dielectric substrate. The frequency range of the double negative response is narrow, which is an inherent character of resonant particles. The particle was fabricated and the results of numerical simulation have been verified experimentally.

The non-bianisotropic version of the resonant particle is composed of an SRR sandwiched between two substrates bearing electric dipoles located on their outer surfaces. Consequently, this structure possesses inversion symmetry.

The metamaterial showing the isotropic response can be finally designed by arranging the presented particles in space with a proper symmetry in a cubic periodic system.

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