A Resonant Particle for a Volumetric DNG Metamaterial

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***Abstract* — This paper studies a combination of electric and magnetic particles in order to obtain simultaneously negative electric and magnetic polarisability under irradiation by an electromagnetic wave. The magnetic particle is represented by a split ring resonator, and electric particle is represented by an electric dipole terminated by an inductor. They are combined together in a way that obtains a resonant element with inversion symmetry. The internal coupling between these resonators located in a waveguide is subjected to a special study. The response of this particle is strongly anisotropic, but an isotropic response can be achieved by a proper spatial arrangement.**

***Key words* —metamaterial, resonant particle, bi-anisotropic, inversion symmetry.**

I. INTRODUCTION

A unit cell of volumetric metamaterials (MTM) for microwaves consists either of a single specifically-shaped element, also known as a particle, or of a suitably spatially-arranged set of particles, all immersed in a host medium [1,2]. For most known applications, an isotropic metamaterial is required. Isotropy can be achieved by periodic arrangement of the inclusions satisfying symmetry of the selected crystallographic group [3-5], or by the use of randomly-oriented inclusions in the volume of the host [6]. However, random arrangements are not easily reproducible and do not show a well-defined electromagnetic response, mainly if the size of the inclusions is not sufficiently small compared to the wavelength, as is usual in most artificial media. Periodic systems are thus usually the preferred option.

Apart from isotropy, another frequent requirement is that the metamaterial will not exhibit magneto-electric coupling, i.e. a non-bianisotropic material is required. In such a case, it is well known [7] that inversion symmetry of the material and thus of the inclusions is needed.

In the present state-of-the-art, there are basically two candidates for the metamaterial defined above. The first candidate is a combination of properly shaped Split Ring Resonators [3] in combination with a 3D wire medium [8]. The second candidate is an arrangement of dielectric or magneto-dielectric spheres [9, 10]. Unfortunately, neither of those two options is easy to make in practice. The problem with the first option is the great complexity of the 3D wire mesh. The second option requires either magneto-dielectric materials or precisely-shaped dielectric objects made from high permittivity materials. In both cases the technological demands are high.

The objective of this paper is to investigate and design a particle that will exhibit enough symmetries for building an isotropic structure, will simultaneously show negative electric and magnetic polarisabilities (DNG), and will at the same time be non-bianisotropic. In addition, we will require that the particle can be manufactured by standard photoetching techniques. This particle is composed as a combination of a split-ring resonator (SRR) and an electric dipole (ED) terminated by an inductor.

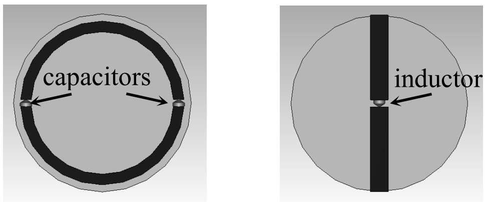
II. ANALYZED AND OPTIMIZED RESONANT PARTICLES

The aim of our work was to obtain a particle showing negative electric and magnetic polarisabilities induced by an exciting electromagnetic wave. At the same time, the particle must have a non-bianisotropic response, i.e., an element with an inversion symmetry. SRR and ED are suitable building blocks for this element. Ceramic substrate with ε_r = 20 was used due to very low losses, as its loss factor is 10^{-4}. The substrate has the shape of a disk 7 mm in radius and 0.4 mm in thickness.

To speed up tedious analysis, simple models of both SRR and ED were used. They are shown in Fig. 1. SRR is composed of a conducting ring cut by two slots terminated by lumped capacitors, Fig. 1a. The radii of SRR are 2 and 2.3 mm. The slot width is 0.2 mm. Capacitance C = 0.55 pF corresponds to the single SRR resonant frequency around 3.1 GHz. ED has the form of a dipole terminated by a lumped inductance, Fig. 1b. To obtain a single ED with resonance at the same frequency of 3.1 GHz as the designed SRR, the following parameters were used: the dipole arms were 2.4 mm in length and 0.4 mm in width. The slot between these arms was 0.2 mm in length. Lumped inductance L = 11.2 nH was used. The thickness of all metal strips was chosen to be 0.1 mm. Metal conductivity was taken relatively low $\sigma = 10^6$ S/m.

The SRR and the EDs are arranged in space in a way that obtains the required inversion symmetry. A model of a possible form of a resonant particle structure is shown in Fig. 2. The SRR is sandwiched between two disk substrates. This sandwiched SRR has resonant frequency 2.82 GHz. Two EDs hosted on separate disk substrates are located parallel to SRR. A special case is a particle with merged substrates, Fig. 2b, which is more suitable for fabrication. To take into account losses in these elements, in addition to metal losses, the capacitors were connected in series with resistors of R_s = 0.1

Ω, and the inductors in parallel with resistors of R_p = 200000 Ω. This structure was analyzed by the CST Microwave Studio when located in the center of an ideal TEM wave waveguide with perfect electric (PEC) walls on top and bottom, and perfect magnetic (PMC) walls on its sides. The waveguide was 20 mm in length, 10 mm in height, and 15 mm in width. This waveguide was filled with air. The analyzed model created in the CST Microwave Studio is presented in Fig. 3. A tetrahedral mesh and a frequency domain solver were used.



a **b**

Fig. 1 Layouts of SRR (a), and of ED (b).

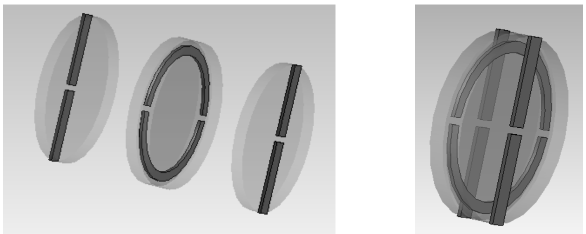


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Fig. 2 Non-bianisotropic DNG resonant particle, with separate substrates (a), and merged substrates (b). The dielectric substrates are made transparent to show the metal strips. Lumped elements are not shown.

III. NUMERICAL RESULTS, DISCUSSION

The particle electric α_{yy}^{ee} and magnetic α_{xx}^{mm} polarisibilities were determined from the scattering parameters, according to the procedure presented in [11]. The resonant element, see Fig. 2, is non-bianisotropic, as it shows inverse symmetry. Then its electric **p** and magnetic **m** dipole moments can be expressed

$$\mathbf{p} = \alpha_{yy}^{ee}\mathbf{E} , \qquad (1)$$

$$\mathbf{m} = \alpha_{xx}^{mm}\mathbf{H} . \qquad (2)$$

Using a simple homogenization procedure [12], applied for elements distributed periodically in space, the effective relative permittivity and permeability, or more precisely the corresponding elements of tensors of these quantities, read

$$\varepsilon_{eff}^{yy} = 1 + \frac{\alpha_{yy}^{ee}}{V\varepsilon_0 - \alpha_{yy}^{ee}/3} , \qquad (3)$$

$$\mu_{eff}^{xx} = 1 + \frac{\alpha_{xx}^{mm}}{V - \alpha_{xx}^{mm}/3} , \qquad (4)$$

where V is the volume of a cell, and ε_0 is vacuum permittivity. A reduced electric polarisability $\alpha_{yy}^{ee}/\varepsilon_0$ is plotted in the following plots (without notification) in order to be directly comparable with the magnetic polarisability value.

The resonant particle shown in Fig. 2 is composed of SRR and ED. These particles influence each other by mutual coupling of a predominantly capacitive character. The closer together the particles are located, the more their particular resonant frequencies are detuned due to this coupling. Consequently, it is necessary to change both inductance L and capacitance C in order to tune the final resonant particle to the original resonant frequency of the single SRR or ED. Only L is changed in this study, with the aim to obtain equal resonant frequencies of SRR and ED, i.e, a single resonant frequency of the designed particle.

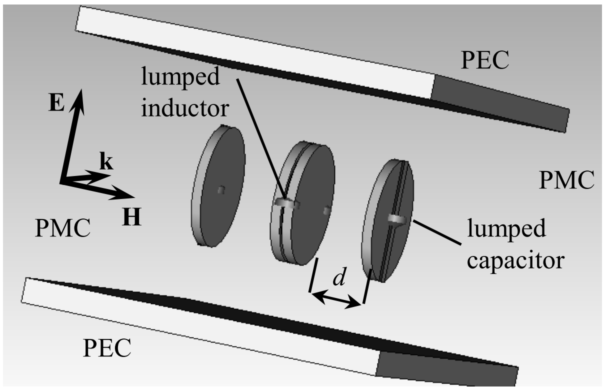


Fig. 3 The CST Microwave model of the resonant element located in the TEM waveguide.

The behavior of the resonant particle, Fig. 2, is studied in dependence on the mutual distance between substrates d, defined in Fig. 3. Two cases for each d are considered in particular. First, the substrate thickness is taken as 0.4 mm, and in the second case the substrates are made thick enough to merge into a single substrate of double thickness, Fig. 2b.

The lowest possible value of the mutual distance is d = 0, which corresponds to the merged substrates, creating a substrate 0.8 mm in total thickness, see Fig. 2b. This will be marked as case 1. In this structure, the best fit of the SRR and ED resonant frequencies was obtained for L = 7.45 nH, and is about 2.76 GHz. Fig 4 shows the calculated real and imaginary parts of the electric and magnetic polarisabilities. The minimal value of $\mathrm{Re}\{\alpha_{yy}^{ee}\}$ here is only -1.27.10^{-7} m^3, and the DNG frequency band is 29 MHz.

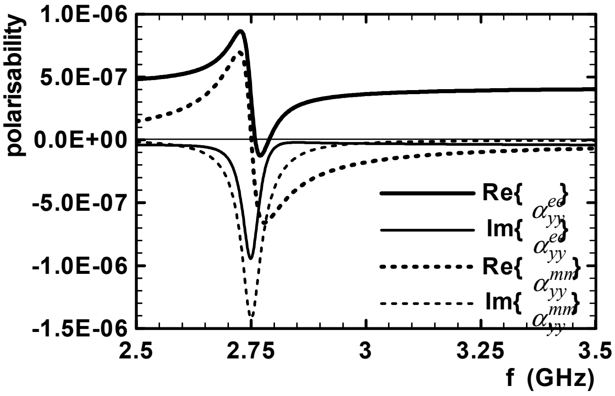


Fig. 4 The electric and magnetic polarisabilities of the resonant particle with substrate distance d = 0, corresponding to the merged substrate 0.8 mm in thickness, case 1.

Increasing d to a value of 0.6 mm, the particle has the shape shown in Fig. 2a, marked as case 2. The resonant frequency can easily be tuned by changing L. The best fit is obtained for value L = 11.3 nH. The resonant frequency is now 2.82 GHz. The polarisabilities are plotted in Fig. 5. Re$\{\alpha_{yy}^{ee}\}$ now reaches minimal value $-1.71.10^{-6}$ m^3, and the frequency band of the DNG response is about 83 MHz.

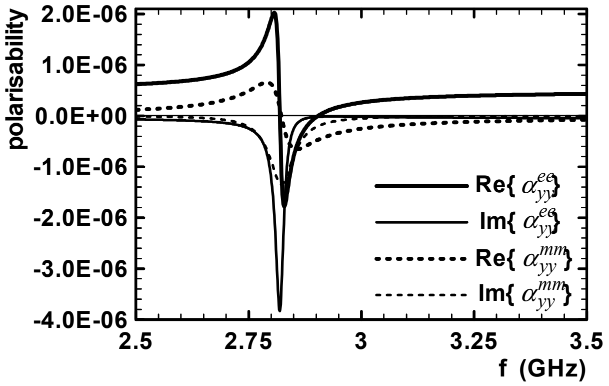


Fig. 5 The electric and magnetic polarisabilities of a resonant particle with substrate distance d = 0.6 mm, case 2.

To obtain a particle with merged substrates, the particular substrate thickness has to be chosen 0.7 mm, The particle has the form from Fig. 2b, and the total merged substrate thickness is 1.4 mm. This structure is marked as case 3. The tuning process for this particle offers the best fit determining the resonant frequency 2.717 GHz at L = 7.39 nH. The corresponding polarisabilities are plotted in Fig. 6. This particle operates as a DNG cell in a narrow frequency band 32 MHz in width. The minimal value of the real part of the electric polarisability is $-1.07.10^{-6}$ m^3.

Let us further increase d to a value of 1.35 mm, and let us retain the particular substrate thickness equal to 0.4 mm. This particle has the form shown in Fig. 2a, and is marked as case 4. The best fit of the resonant frequencies of the SRR and the ED of this particle is obtained for value L = 12.05 nH, resulting in resonant frequency 2.82 GHz and giving a DNG frequency band equal to 108 MHz. This is similar to case 2 from Fig. 5. The minimum of Re$\{\alpha_{yy}^{ee}\}$ is $-1.93.10^{-6}$ m^3. A merged substrate 2.15 mm in total thickness, case 5, is obtained by filling the whole space between the particular substrates from case 4 with a ceramic material. The tuning process has now been finished by L = 7.65 nH, assuring resonant frequency 2.682 GHz and a DNG band 34 MHz in width, similar to the behavior shown in Fig. 6 for case 3. The minimum of the electrical polarisability real part is $-1.36.10^{-6}$ m^3.

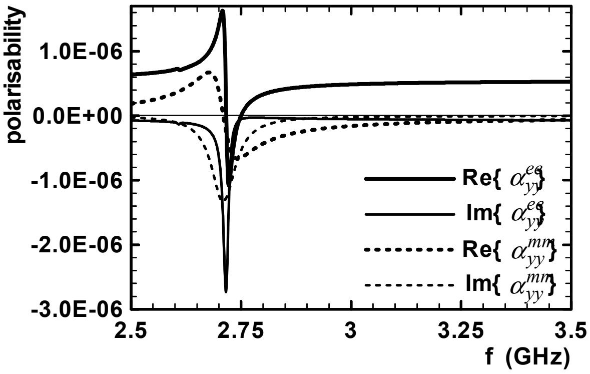


Fig. 6 The electric and magnetic polarisabilities of a resonant particle with merged substrates 1.4 mm in total thickness, case 3.

The real parts of the reduced electric polarisabilities of cases 2, 3, 4, and 5 are plotted in Fig. 7, clearly showing their difference. The magnetic response of these resonant particles differs not very significantly from case to case. The widest frequency band of DNG behavior was obtained for the resonant structure marked case 4, with the most distantly placed SRR and EDs. Here the resulting resonant frequency equals the original resonant frequency of the sandwiched SRR 2.82 GHz. In structures with merged substrates (cases 1, 3, 5), the additional dielectric material causes stronger coupling, and the DNG effect becomes weaker with reducing distance between SRR and ED.

There is a very significant influence of the level of the losses on the particle response. This is documented by the plots in Fig. 8 applied for a particle with the case 1 structure. High losses precludes the resonant particle from the DNG response, as is evident from Fig. 8, lines (a) and (b).

The effective relative permittivity and permeability are calculated from the polarisabilities, using (3) and (4). Their values depend on the volume of the cell in which the resonant particle is located, and on the loss level. For a cubic cell with edge 10 mm in length, the effective real parts of the parameters ε_{eff} = -1.5, and μ_{eff} = -1.55 were obtained at frequency 2.815 GHz and with DNG band 48 MHz for the particle marked as case 4, with the losses defined in Fig. 9 lines (d), (e).

For the design and fabrication of the particle, SRR can be simply used according to Fig. 1a with lumped capacitors. For ED, the structures presented, e.g., in [13. 4] can be used, Fig. 9.

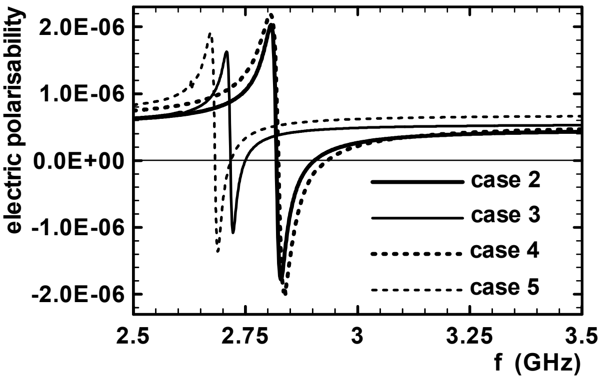


Fig. 7 The real part of the electric polarisability of the four resonant particles. The lines for cases 2 and 3 are taken from Figs. 5 and 6.

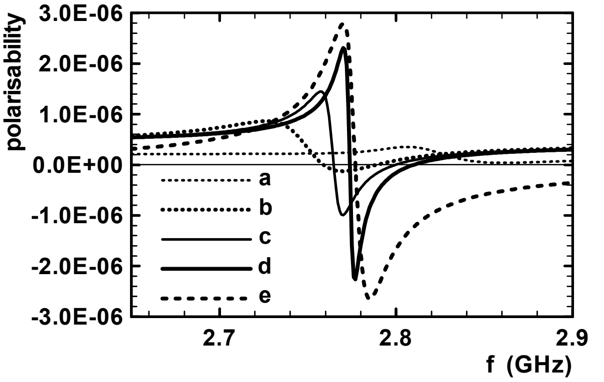
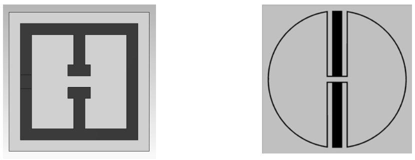


Fig. 8 The real parts of the polarisabilities of the particle denoted in the text as case1. The lines are: electric polarisability for $R_s = 0.5\ \Omega$, $R_p = 10000\ \Omega$, $\sigma = 10^6$ S/m (a), for $R_s = 0.1\ \Omega$, $R_p = 200000\ \Omega$, $\sigma = 10^6$ S/m (b), for $R_s = 0\ \Omega$, $R_p = 2.10^7\ \Omega$, $\sigma = 10^6$ S/m (c), for $R_s = 0\ \Omega$, $R_p = 2.10^7\ \Omega$, all metals PEC (d), magnetic polarisability (e) as for line (d).

VII. CONCLUSIONS

This paper presents a resonant particle with DNG behavior and a non-bianisotropic response suitable for the design of volumetric metamaterials. This particle is composed of an SRR sandwiched between two disk substrates and placed between two electric dipoles located on their own substrates. Two versions of this particle were investigated. The first version has separately located SRRs and EDs. This particle has a wider frequency band of the DNG response and is more sensitive to the electric field than the second particle with the merged substrates. This behavior is, however, significantly influenced by the level of losses in the structure. The DNG response gradually disappears with increasing losses. Simplified SRR and EDs with lumped capacitors and inductors were used. In practical realization, real planar resonators with adequate structures must be applied.



a **b**

Fig. 9 Usable planar structures replacing the ED model in fabricated resonators, taken from [13] (a), and from [4] (b).

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