A System of Split Ring Resonators with Randomly Dispersed Resonance Frequencies

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***Abstract* - This paper reports on a 3D system of broad side coupled split ring resonators with randomly dispersed resonance frequencies. This dispersion is caused by the varied geometric and material parameters of the resonators. The resonance frequency band of the effective permeability of the metamaterial is significantly widened, the value of the real part of the permeability is reduced, or the band of negative permeability even disappears. The paper comments on the design of a lens aimed at improving the imaging properties of the MRI system. The proposed homogenization method is verified by measuring the scattering parameters of the metamaterial in a rectangular waveguide.**

Index Terms — Metamaterial, homogenization, effective permeability, split ring resonator, perfect lens, MRI.

I. Introduction

Metamaterials with negative permeability are designed as 3D periodic systems of resonant particles showing negative magnetic polarizabilities. Variations in the material and geometric parameters of resonant particles can be treated by accounting resonators with dispersed resonance frequencies [1]. This widens the response of the metamaterial and reduces the values of its effective parameters. The band of negative effective permeability can even disappear.

Only limited attention has been paid in the literature to investigations of metamaterials with randomly positioned resonant particles or with randomly dispersed material or geometrical parameters. Systems of randomly oriented chiral particles were studied in [2]. The transmission properties of waveguides with statistical disorder in their periodicity were studied in [3]. A fully experimental investigation of randomness in metamaterials was performed in [4-8]. Papers [1, 9] investigated 3D periodic system of SRRs. Fluctuations of SRR parameters were represented by the fluctuations of their resonance frequencies. The authors showed that microscopic averaging can be performed prior to statistical averaging only under the assumption that the resonance frequency is a slowly varying random function of the coordinates.

This work studies the behavior of a 3D periodic system of broad side coupled SRRs (BC-SRR) aligned in one direction. This periodic system is homogenized using the way presented by Tretyakov [10], assuming that particular magnetic dipoles have randomly distributed resonance frequencies. The results of the analysis are verified by measuring the transmission of the system of BC-SRRs inserted in a waveguide. Finally, we comment on the procedure for designing a lens used in the MRI system [11], assuming that the resonators without perfectly equal parameters are used.

II. Homogrenization Procedure

Let us assume a system composed of BC-SRRs located in a 3D periodical net with period a aligned in the z direction. Period a is assumed to be much smaller than the free space wavelength. The resonators are approximated by point magnetic dipoles with polarizability [12]

$$\alpha(\omega,\omega_{oi}) = \frac{A\omega^2}{\omega_o^2 - \omega^2 + j\omega\delta}\ , \qquad (1)$$

where ω and ω_o are angular and resonance angular frequencies, respectively, and A is amplitude. The total averaged (macroscopic) magnetic field $\hat{\mathbf{H}} = \hat{H}\,\mathbf{z}_o$ is assumed to be aligned with the orientation of the dipole. All fields are averaged over the cell volume $V = a^3$. The magnetic field exciting the reference magnetic dipole located at the origin of the coordination system can be written as in [16]

$$H_z^{(\mathrm{ind})} = \hat{H}_z + \sum_{i\neq 0}\left(H_{iz} - \hat{H}_{iz}\right) - \hat{H}_{0z} = \hat{H}_z + \sum_{i\neq 0} m_i\left(F_i - \langle F_i\rangle\right) - \hat{H}_{0z} \qquad (2)$$

Field $H_{i\,z}$ is the field at the position of the reference dipole excited by a point magnetic dipole located at position i, and can be expressed as in [13] eq. (9.35). In (2), these fields are summed over all magnetic dipoles, with the exception of $i = 0$, as the dipole cannot excite itself. The average value of the field excited by the reference magnetic dipole is [13]

$$\hat{H}_{0z} = \frac{2}{3a^3} m_0\,. \qquad (3)$$

This is exactly valid for a stationary field, but is acceptable here at the low frequency limit.

Magnetic field (2) defines the magnetic moment of the reference magnetic dipole. It is assumed that all magnetic dipoles are excited by the same field. Inserting (3) into (2), one can get the magnetizing field $H_z^{(ind)}$

$$H_z^{(\mathrm{ind})} = \frac{\hat{H}}{1 + \dfrac{2}{3a^3}\alpha_0(\omega,\omega_o) - \sum_{i\neq 0}\alpha_i(\omega,\omega_{oi})\left(F_i - \langle F_i\rangle\right)}\,. \qquad (4)$$

The effective permeability is determined using the relation

$$B = \mu_o\left(\hat{H} + M\right) = \mu_o\left(\hat{H} + \frac{m_0}{a^3}\right) , \quad (5)$$

$$\mu_{zz}(\omega,\omega_o) = 1 + \\ + \frac{\alpha_0(\omega,\omega_o)}{a^3\left(1 + \frac{2}{3a^3}\alpha_0(\omega,\omega_o) - \sum_{i\neq 0}\alpha_i(\omega,\omega_{oi})\left(F_i - \langle F_i\rangle\right)\right)} , \quad (6)$$

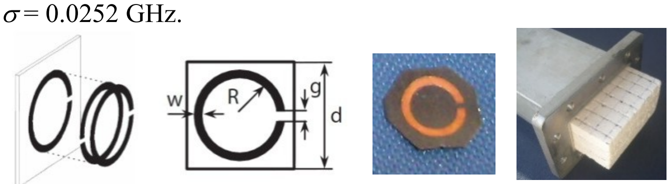
Due to the symmetry of the system, all other components of the permeability tensor are equal to zero.

In the summation of (6), the resonance frequencies are randomly chosen around the central value, independently for each magnetic dipole, and (6) is averaged over a chosen number of realizations. This averaging replaces N dimensional integration. The resulting process has fast convergence. In this way, the randomness of resonant particles is built into the homogenization process, as stated in [9].

III. Details of the Analysis and the Analyzed Structure

The procedure presented in the previous paragraph is applied using the following details to get convergent results. The summation in (6) is taken over the cube centered at the origin of the coordinate system, with its edge equal to $4a$. The resonance frequencies of particular current loops are randomly selected according to their normal distribution defined below. The final sums are averaged over 300 runs.

The analyzed periodic system is composed of BC-SRR, see Fig. 1. The resonators are fabricated on a Rogers RT Duroid 5880 substrate 0.127 mm in thickness with permittivity 2.2 and 0.017 mm copper cladding. The dimensions according to Fig. 1a are: $R = 1.8$ mm, $w = 0.7$ mm, $g = 0.3$ mm, $d = 7$ mm. The procedure for determining the free space polarizability presented in [14] provided the following parameters: $\omega_0 = 1.922.10^{10}$ s^{-1}, $A = 5.10^8$ m^3, $\delta = 5.5.10^7$ s^{-1}, defined in (1). The resonance frequencies of 149 BC-SRRs were measured in order to determine their distribution function. This function was fitted by a Gaussian distribution with mean value $\omega_0 = 3.06$ GHz, and with standard deviation $\sigma = 0.0252$ GHz.



(a) **(b)** **(c)**

Figure 1. Planar broad-side coupled split ring resonators: a sketch of the layout (a), fabricated resonator (b), a measured metamaterial prism (c).

The metamaterial used for the experiment is a prism of dimensions 70×70×33 mm, assembled by inserting BC SRRs, see Fig. 1b, into three slices of polystyrene 11 mm in thickness with period $a = 11$ mm, see Fig. 1c.

A change to the value of structure period a changes the mutual coupling between particular resonators and detunes the resonance frequency of the metamaterial. This frequency decreases with increasing a. The effect of negative permeability disappears at sufficiently high a. For the structure analyzed here, this happens for $a > 20$ mm. As period a decreases, tighter coupling raises the maxima of the real part of the effective permeability, and the losses represented by the imaginary part increase.

IV. BC-SRR with Randomly Dispersed Resonance Frequencies

The behavior of the metamaterial defined above with dispersed resonance frequencies is documented in Fig. 2. Dispersion curves for $a = 11$ mm are plotted here in dependence on the standard deviation value. Increasing dispersion of the resonance frequencies of the particles reduces the absolute value of the effective permeability and widens the frequency band around resonance. For standard deviation greater than 0.035 GHz the band of negative effective permeability disappears. The negative maximum of the imaginary part of the effective permeability increases with decreasing standard deviation.

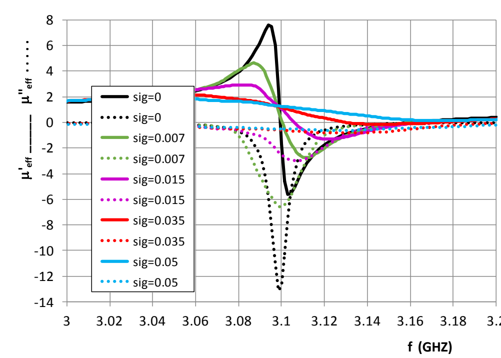


Figure 2. Calculated effective permeability of the metamaterial with period $a = 11$ mm in dependence on the standard deviation, shown in GHz.

The scattering parameters of the metamaterial prism defined above were measured by a VNA in a rectangular waveguide with metallic walls R32 in order to verify the theory presented here. The measurement results were compared with the scattering parameters, calculated by the CST Microwave Studio, applying the complex permeability determined by the homogenization process presented above. The data are plotted in Fig. 3. The curve denoted CST S21 represents the analysis of the metamaterial with the distribution of resonance frequencies determined by the standard deviation 0.025 GHz.

V. MRI Lens

The final example of the analysis presents the behavior of a metamaterial made of resonant loops terminated by capacitors aimed at improving the resolution of the lens for the 1.5 T MRI system working at 63.87 MHz [11]. The resonance of the loops used here is 0.0554 GHz. The polarizability of the resonant particle is defined by the parameters used in (1):

$\omega_0 = 3.48.10^8$ s^{-1}, $A = 7.1.10^{-7}$ m^3, $\delta = 1.10^7$ s^{-1}. The period of the 3D system is 15 mm. Assuming resonators without dispersion of the resonance frequencies, the metamaterial is designed to show the real part of the effective permeability equal to -1 at 63.87 MHz, with a relatively low imaginary part $\mu''_{eff} = -0.5$. The sensitivity of the real part of the effective permeability to the parameters of the structure is rather small at the working frequency. Fig. 4 shows that the lens can be designed using elements with the resonance frequency distribution defined by resonance frequency 0.0554 GHz and by standard deviation up to 1.5 MHz. The real part of the effective permeability still reaches a value of -1 around a frequency of 0.0635 GHz at this dispersion.

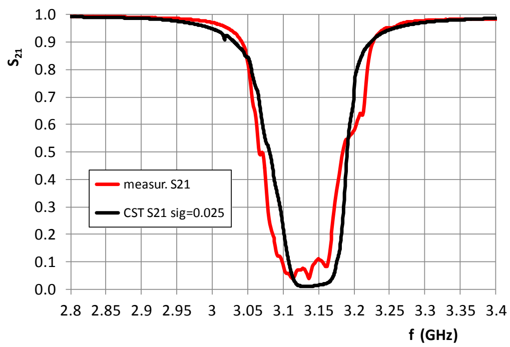


Figure 3. Measured and calculated scattering parameters of a metamaterial prism with period a = 11 mm.

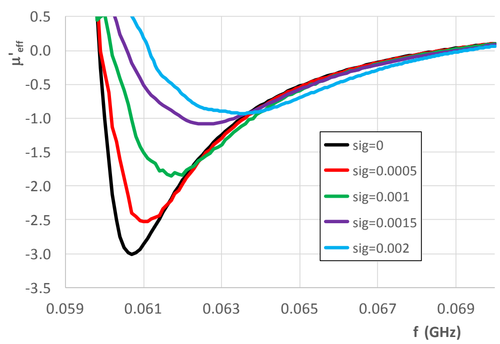


Figure 4. Calculated real part of the effective permeability of the metamaterial for an MRI lens with period a = 15 mm

VI. Conclusion

The randomness caused by the dispersion of the resonance frequencies is incorporated into the process of metamaterial homogenization. In this process, BC-SRRs are represented by point magnetic dipoles. The complex effective permeability of the metamaterial is calculated. The homogenization process presented here works effectively, with relatively fast convergence.

The random distribution of the resonance frequencies of particular current loops - BC-SRRs - significantly disperses the resonance frequency band of the effective permeability of the metamaterial. At the same time, it reduces the magnitude of the real part of the effective permeability, and the frequency band of its negative value is narrowed, or at higher values of the standard deviation of the resonance frequencies this band even disappears. This effect is verified experimentally by measuring the scattering parameters of the metamaterial block in a rectangular waveguide, and comparing these results with the parameters calculated by CST Microwave Studio, using calculated complex permeability dispersion.

Finally, the homogenization procedure presented here is used to determine the limit in which the dispersion of the resonance frequency of particular resonators can be varied in order to obtain a working lens designed to improve the quality of the images offered by the 1.5 T MRI system working at 63.87 MHz.

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