A Polarizability Measurement Method for Electrically Small Particles

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***Abstract*—A simple and precise free-space polarizability measurement method for electrically small particles is presented. The method is based on measuring the scattering parameters of a waveguide loaded by a particle and on knowledge of the dipolar polarizabilities of two calibration standards. The method is verified on a metallic disc and a resonant conducting ring terminated by a capacitor.**

***Index Terms*—Artificial materials, electromagnetic measurements, polarizability.**

I. Introduction

The SCATTERING properties of electrically small bodies have always been of immense importance, both for theory and practical applications. Particularly, the dipolar polarizability and the geometrical structure of a lattice are the two key properties of any material that enable unique determination of the constitutive parameters, i.e., permittivity and permeability. At the same time, dipolar polarizability is uniquely related to the radar cross section (RCS) of the scatterer [1], which is a major characteristic of radio frequency identification (RFID) tags.

In canonical cases, there exist analytical models for polarizability [2], [3]. However, during the development of metamaterials [4] and chipless RFID [5], the geometry of the scaterrers becomes complex, and the polarizability of realistic particles can only be extracted through numerical methods [6]. When accompanied by fabrication imprecision and incomplete knowledge of material properties, the polarizability can only be retained by measurement. Unfortunately, few methods for measuring polarizability have been proposed so far. Pioneering work has been done by Cohn [7]. Unfortunately, a particle is placed in an electrolyte in his method, and this is not applicable at high frequencies and for resonant or active particles, which will not work inside an electrolyte. Two more methods have been proposed recently. The first [8], [9] obtains the particle polarizability from a measurement of the scattering parameters of a waveguide segment loaded by the particle, while the second method [10] uses a measurement of the scattering parameters of a two-dimensional square array of identical particles. These two methods are in fact equivalent if a square TEM waveguide composed by two electric and two magnetic walls is used.

In this letter, we propose and experimentally verify a generalization of the waveguide measurement method [9]. Unlike the original method, our new method is free of any knowledge of the waveguide geometry, its modes, and their coupling with the analyzed particle. This is effectively eliminated by proper calibration, using two particles of known polarizability. For this purpose, a metallic disc and a metallic sphere are used.

Manuscript received February 18, 2014; revised May 07, 2014; accepted May 26, 2014. Date of publication May 29, 2014; date of current version June 11, 2014. This work was supported by the Czech Science Foundation under Project No. 13-09086S and the Czech Technical University in Prague Project No. SGS13/198/OHK3/3T/13.

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Digital Object Identifier 10.1109/LAWP.2014.2327152

II. Polarizability Extraction

Let us assume a particle that is small in comparison to the used wavelength. The interaction of this particle with an electromagnetic field is well described by the induced dipole moments. In the following, we will assume that the particle is excited by the fields $E_{\rm loc}$, $H_{\rm loc}$, which are, in a good approximation, homogeneous in its volume. In addition, we will assume that the particle has negligible cross polarization, i.e., that the induced dipoles are colinear with the exciting fields. In this case, the induced dipole moments are given by

$$\begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} \alpha^{\rm ee} & \mu_0\alpha^{\rm em} \\ \alpha^{\rm me} & \mu_0\alpha^{\rm mm} \end{bmatrix} \begin{bmatrix} E_{\rm loc} \\ H_{\rm loc} \end{bmatrix} \tag{1}$$

where $\alpha^{\rm ee}$, $\alpha^{\rm mm}$ are the electric and magnetic polarizabilities and where $\alpha^{\rm em}$, $\alpha^{\rm me}$ are the magneto-electric polarizabilities, respectively. If this particle is inserted into a mono-mode waveguide (all higher-order modes are evanescent), it will be excited and will scatter the impinging wave. If the dominant mode of the waveguide is such that the exciting fields are mostly transversal in the volume occupied by the particle (all TEM waveguides, center of TE_{10} metallic waveguide, etc.), then

$$\begin{bmatrix} E_1^- \\ E_2^- \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} E_1^+ \\ E_2^+ \end{bmatrix} + \begin{bmatrix} C_p & -C_m \\ C_p & C_m \end{bmatrix} \begin{bmatrix} p \\ m \end{bmatrix} \tag{2}$$

where index $+$ represents a wave impinging on the particle, while index $-$ represents the scattered wave (reference planes are positioned at the particle). The subscripts 1, 2 represent the field in front and behind the particle, respectively. The coefficients C_p and C_m characterize the conversion of the dipole moments into the field of the dominant mode, and they generally depend on the geometry of the waveguide, the used mode and the angular frequency ω. The local fields acting on the particle can now be quite generally written as

$$\begin{bmatrix} E_{\rm loc} \\ H_{\rm loc} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ Y & -Y \end{bmatrix} \begin{bmatrix} E_1^+ \\ E_2^+ \end{bmatrix} + \bar{\mathbf{C}}_{\rm int} \begin{bmatrix} p \\ m \end{bmatrix} \tag{3}$$

where Y is the modal admittance of the dominant mode and matrix $\bar{\mathbf{C}}_{\rm int}$ is composed of interaction constants that reflect the environment around the particle. Combining (1)–(3), we can arrive at (4), shown at the bottom of the page, where $s_{11}, s_{12}, s_{21}, s_{22}$ are the measured scattering parameters with reference planes at the center of the particle and where following normalization:

$$\alpha_{\rm n}^{\rm ee} = \frac{\alpha^{\rm ee}}{\varepsilon_0 V} \qquad \alpha_{\rm n}^{\rm mm} = \frac{\mu_0 \alpha^{\rm mm}}{V}$$
$$\alpha_{\rm n}^{\rm em} = Z_0 \frac{\alpha^{\rm em}}{V} \quad \alpha_{\rm n}^{\rm me} = Z_0 \frac{\alpha^{\rm me}}{V} \tag{5}$$

has been used, with V being a given reference volume and Z_0 being the free-space impedance. Equation (4) also contains new dimensionless (but as yet unknown) constants $C_{11}, C_{12}, C_{21}, C_{22}$, and $\bar{\mathbf{C}}_{\rm n}$.

It follows from (4) that any two particles with known polarizabilities fully characterize all unknown constants and can be used as standards. In the following, we will use the known polarizabilities of an electrically small perfectly conducting sphere [2], [3]:

$$\frac{1}{\alpha^{\rm mm}\mu_0} = -\frac{1}{2\pi r^3} + {\rm j}\frac{k_0^3}{6\pi}$$
$$\frac{\varepsilon_0}{\alpha^{\rm ee}} = \frac{1}{4\pi r^3} + {\rm j}\frac{k_0^3}{6\pi} \tag{6}$$

and of an electrically small infinitesimally thin perfectly conducting disc (oriented so that the magnetic field is perpendicular to its surface) [2], [3]

$$\frac{1}{\alpha^{\rm mm}\mu_0} = -\frac{3}{8r^3} + {\rm j}\frac{k_0^3}{6\pi}$$
$$\frac{\varepsilon_0}{\alpha^{\rm ee}} = \frac{3}{16r^3} + {\rm j}\frac{k_0^3}{6\pi} \tag{7}$$

where r is the radius of the sphere or of the disc and k_0 is the free-space wavenumber. Note, however, that both the sphere and the disc are non-bianisotropic ($\alpha^{\rm em} = 0, \alpha^{\rm me} = 0$), which leads to $s_{11} = s_{22}$ and $s_{12} = s_{21}$ during the calibration. Consulting such conditions with (4) implies that such calibers cannot characterize the magneto-electric (off-diagonal) constants. With such calibration, the method should thus coherently be used only for non-bianisotropic particles. However, having calibers with analytically known bianisotropic polarizabilities, the method could be used in its full extent.

III. Results

The polarizability measurement method described in the above paragraph has been applied to the rectangular R32 (WG10) waveguide (length $\approx$ 700 mm) fed by coaxial–waveguide transitions and loaded at its center with an analyzed particle. Prior to the use of the extraction method, the TRL calibration with reference planes at the center of the particle was utilized. The metallic disc and metallic sphere of radius $r = 10$ mm were used as the two polarizability standards; see Fig. 1. The normalization volume was advantageously chosen as $V = r^3$. Using (4) over the measured scattering parameters of a waveguide loaded by the sphere and the disc of radius r, the unknown constants $C_{11}, C_{12}, C_{21}, C_{22}$, and $\bar{\mathbf{C}}_{\rm n}$ were found and were used for subsequent measurements.

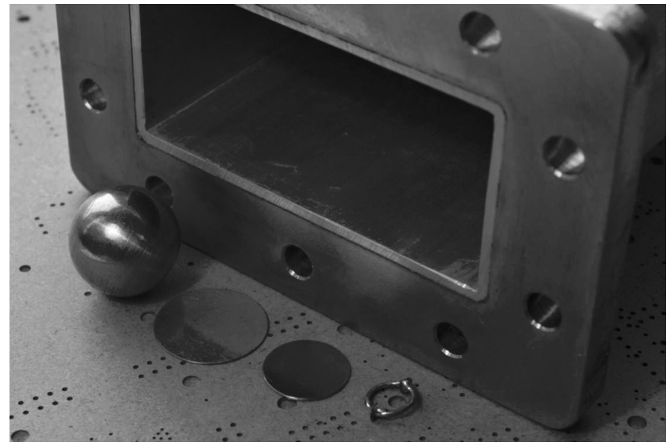

Fig. 1. Photograph of the R32 (WG10) waveguide, calibration metallic sphere of radius 10 mm, calibration metallic disc of radius 10 mm, measured metallic disc of radius 7.5 mm, and measured resonant ring loaded by two parallel-plate capacitors.

With respect to the calibration process, it is important to remember that the method assumes the polarizability standards and the measured particle to be placed at the same position in the waveguide, i.e., it assumes that the constants $C_{11}, C_{12}, C_{21}, C_{22}$, and $\bar{\mathbf{C}}_{\rm n}$, and the reference planes are the same for all three measurements. A deviation from this ideal situation will introduce a systematic error, and it is important to stress that the method is quite sensitive to it. The most critical is the precise positioning of the reference planes. The dependence on the lateral misplacement is not so strong since the field is assumed to be homogeneous in the volume of the particle and the coupling with the walls of the waveguide is usually weak.

A metallic disc of radius 7.5 mm was used as a first testing object; see Fig. 1. Its measured electric and magnetic polarizabilities are shown in Figs. 2 and 3 and are compared to the analytical prescription (7) and to the CST simulation of the described waveguide scenario. The precision in this case is greatly affected by the very low reflectivity of the measured object. Nevertheless, the extracted results can be considered as satisfactory. The observed mismatch between the CST simulation and the analytical model should be attributed to a systematic error of the method, most probably connected to the inhomogeneity of the fields of the $\rm TE_{10}$ mode and possible slight misplacement of the polarizability standards and the measured object during the calibration process.

In a second test, the magnetic polarizability of a capacitively loaded metallic ring was measured; see Fig. 1. The ring was made of copper wire ($\sigma \approx 5.6 \cdot 10^7$ S/m) of circular cross section with radius $r_0 = 0.5$ mm. The mean radius of the ring was $a = 3.9$ mm. The ring was loaded in each of the two centrally symmetric gaps by a parallel-plate capacitor, which was designed as a rectangle (2×2 mm^2) of metallized dielectric

$$\begin{bmatrix} \alpha_{\rm n}^{\rm ee} & \alpha_{\rm n}^{\rm em} \\ \alpha_{\rm n}^{\rm me} & \alpha_{\rm n}^{\rm mm} \end{bmatrix}^{-1} = \frac{\begin{bmatrix} C_{11}\left(2 + s_{11} + s_{22} - s_{12} - s_{21}\right) & C_{12}\left(s_{11} - s_{22} + s_{21} - s_{12}\right) \\ C_{21}\left(s_{22} - s_{11} + s_{21} - s_{12}\right) & C_{22}\left(2 - s_{11} - s_{22} - s_{12} - s_{21}\right) \end{bmatrix}}{s_{11}s_{22} - s_{12}s_{21} + s_{21} + s_{12} - 1} + \bar{\mathbf{C}}_{\rm n} \tag{4}$$

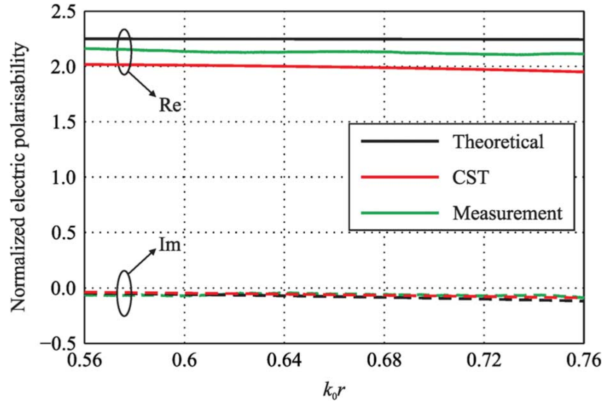

Fig. 2. Normalized electric polarizability of a metallic disc of radius 7.5 mm.

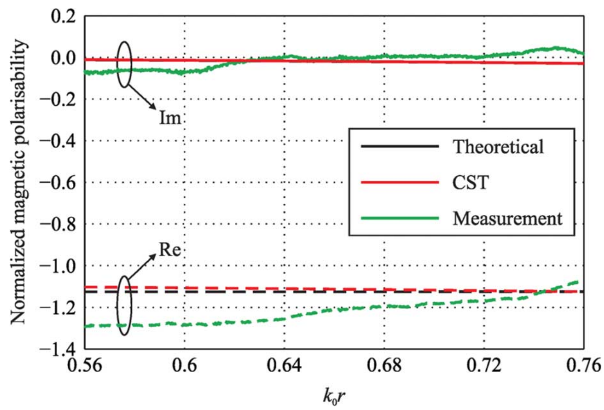

Fig. 3. Normalized magnetic polarizability of a metallic disc of radius 7.5 mm.

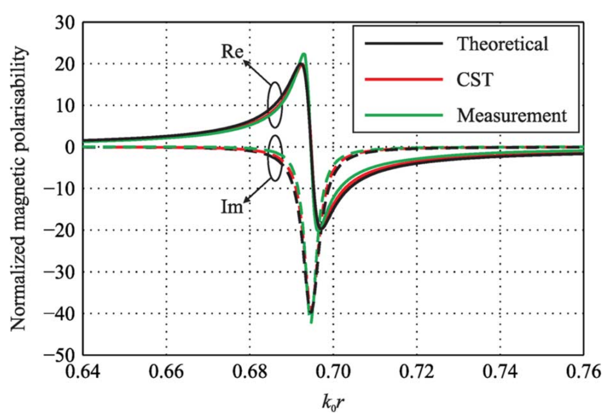

Fig. 4. Normalized magnetic polarizability of a metallic ring loaded by a parallel-plate capacitor.

substrate of thickness 0.25 mm, permittivity 2.3, and dielectric loss $\mathrm{tg}\delta \approx 0.0015$. The measured magnetic polarizability is depicted in Fig. 4, together with the CST simulation of the same scenario and an analytical model [3]

$$\frac{V}{\mu_0 \alpha^{\mathrm{mm}}} = \left(\frac{V}{a^3}\right)\left(\frac{L}{\mu_0 \pi^2 a}\right)\left(\mathrm{j}\frac{R_{\mathrm{met}} + R_{\mathrm{rad}}}{\omega L} + \frac{\omega_0^2}{\omega^2 (1 - \mathrm{jtg}\delta)} - 1\right) \quad (8)$$

where ω_0 is the resonant frequency, $R_{\mathrm{met}} = (a/r_0)\sqrt{(\omega\mu_0)/(2\sigma)}$ is the resistance due to finite conductivity, and $R_{\mathrm{rad}} = Z_0 \pi k_0^4 a^4/6$ is the radiation resistance of a loop. The inductance of the ring L can be estimated as [11]

$$L = a\,\mu_0 \left[\ln\left(\frac{8a}{r_0}\right) - 2\right]. \quad (9)$$

For the purposes of comparison, the capacitor used in the numerical simulation (CST) and in the analytical model was slightly modified in order to precisely fit the experimentally observed resonance frequency. Other parameters of the models were unaltered. Neglecting the possible frequency shift, the correspondence between the theory, numerical simulation, and measurement can be considered as excellent.

IV. Conclusion

In summary, we have developed a simple method that can potentially serve for measuring free-space dipolar polarizabilities. The method is based on knowledge of the polarizabilities of two objects (standards) that perform the calibration. For the purposes of verification, the method was calibrated by a metallic disc and a metallic sphere, and was then used to measure the polarizability of another metallic disc and a resonant ring loaded by two capacitors. The extracted polarizabilities were compared to values calculated analytically and numerically by CST Microwave Studio, reaching very good agreement and validating the method. The method is a very promising candidate for an experimental characterization of RCS of electrically small scatterers, for which free-space methods are very problematic to use.

Acknowledgment

The authors would like to thank M. Prihoda, V. Adler, and M. Haase from the Department of Electromagnetic Field at Czech Technical University in Prague for their help with the measurements.

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