Utilization of Symmetries for MIMO Systems

Michal Mašek, Miloslav Čapek, and Lukáš Jelínek

Department of Electromagnetic Field,
Czech Technical University in Prague,
Prague, Czech Republic
michal.masek@fel.cvut.cz

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Outline

Goal

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  - Usable, for example, in MIMO systems.

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- What we understand by uncorrelated?

$$p\left(f_m, f_n\right) = \left\langle f_m, \mathcal{A}\left(f_n\right)\right\rangle = 0 : \; f_m \in \mathcal{S}, \; f_n \in \mathcal{T}$$

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Used tools

- Method of Moments[1].
- Point group theory[2].

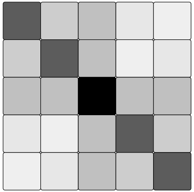
[1] R. F. Harrington, *Field Computation by Moment Methods*. Piscataway, New Jersey, United States: Wiley – IEEE Press, 1993

[2] R. McWeeny, *Symmetry: An Introduction to Group Theory and Its Applications*. London: Pergamon Press, 1963

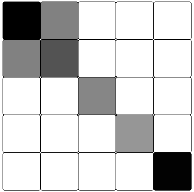
Block-Diagonalization of the Matrix

- Operator $\mathcal{A}$ is represented by matrix $\mathbf{A}$.
- Block-diagonalization:

$$\widehat{\mathbf{A}} = \mathbf{\Gamma}^{\mathrm{T}} \mathbf{A} \mathbf{\Gamma}$$



Full impedance matrix $\mathbf{A}$.



Block-diagonalized impedance matrix $\widehat{\mathbf{A}}$.

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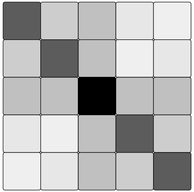
Generality

- All operators $\mathcal{A}$ belonging to a given antenna setup are block-diagonalized at the same time!



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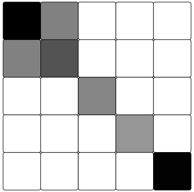

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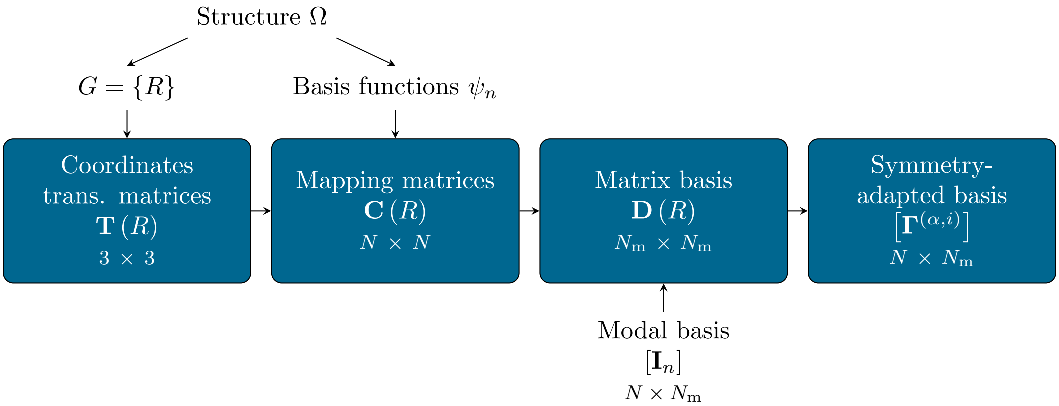
Numerical Procedure[3]

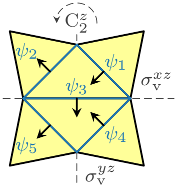


[3] M. Masek, M. Capek, L. Jelinek and K. Schab, "Modal Tracking Based on Group Theory," submitted to *IEEE Transactions on Antennas and Propagation*, available online at ArXiv:1812.03006, https://arxiv.org/abs/1812.03006

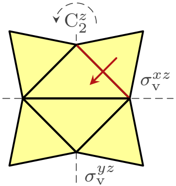
Symmetry-Adapted Vectors[2,4]

$$\mathbf{V}^{(\alpha,i)} = \frac{g^{\alpha}}{g} \sum_{R \in G} \widetilde{d}_{ii}^{\alpha}(R)\, \mathbf{C}(R)\, \mathbf{v}$$

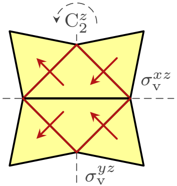
Example: point group[5] C_{2v}.



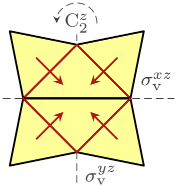
Orientation of basis function.



$\mathbf{v} = [1, 0, 0, 0, 0]^{\mathrm{T}}$.



$\mathbf{V}^{(1)} = [1, 1, 0, 1, 1]^{\mathrm{T}}$.



$\mathbf{V}^{(2)} = [1, -1, 0, 1, -1]^{\mathrm{T}}$.

[2] R. McWeeny, *Symmetry: An Introduction to Group Theory and Its Applications*. London: Pergamon Press, 1963

[4] α and i are point group indices to distinguish between sets.

[5] Point group C_{2v} consists of 4 symmetry operations and has 4 one-dimensional irreducible representations.

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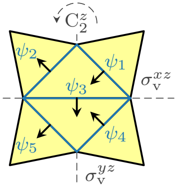
- Taking $\mathbf{v} \in \mathbb{1}$ leads to matrix $\boldsymbol{\rho}^{(\alpha,i)}$.

Example: point group[5] $\mathrm{C}_{2\mathrm{v}}$.



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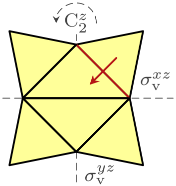

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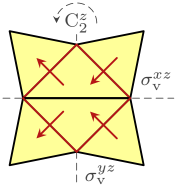

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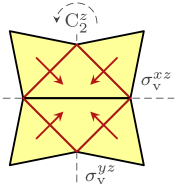

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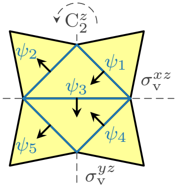
Example: point group[5] C_{2v}.

- Taking $\mathbf{v} \in \mathbb{1}$ leads to matrix $\boldsymbol{\rho}^{(\alpha,i)}$.
- Linear independent vectors from $\boldsymbol{\rho}^{(\alpha,i)}$ form rectangular matrix $\boldsymbol{\Gamma}^{(\alpha,i)}$.



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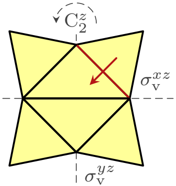

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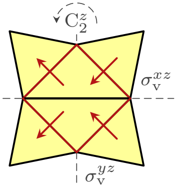

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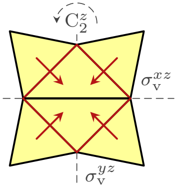

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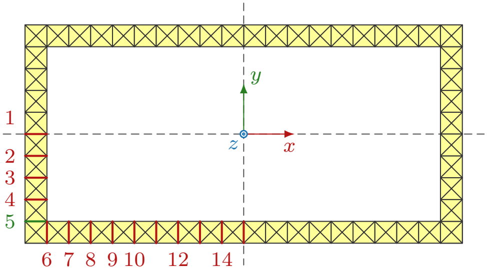
Example – The Structure

- Rectangular loop with 2:1 edge length ratio and with electrical size $ka = 2$.
- Symmetry point group $\mathrm{C}_{2\mathrm{v}}$.
  - Four one-dimensional irreducible representations,
  - *i.e.*, four channels can be found.

Brute-force method

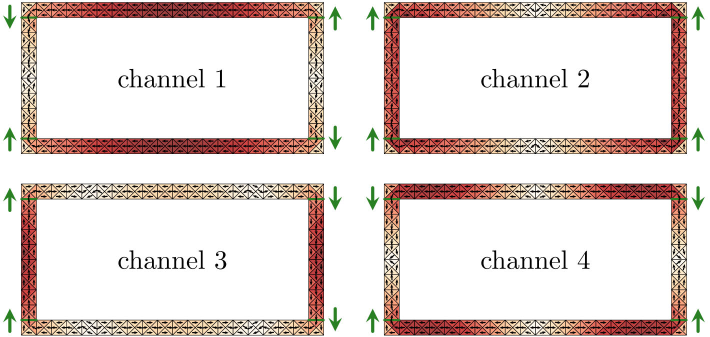
Each basis function is tested one by one for all four channels.

- Restriction only on non-diagonal BFs.
- Only 1/4 of the object is tested.
  (BFs are higlighted by red color.)



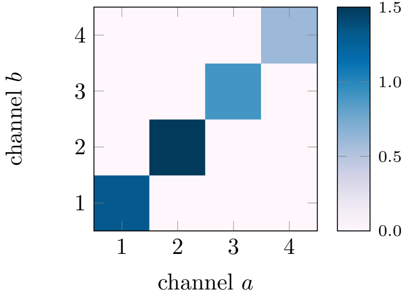
Mesh of the rectangular loop.

Example – Current Distributions



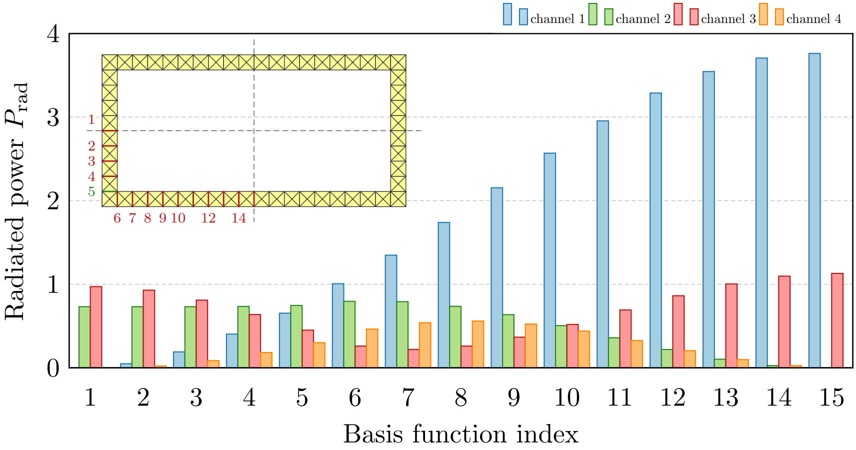
Normalized total current densities of each channel. Feeders are depicted by green lines, arrows define the direction of the unitary voltage source.

$$P_{\mathrm{rad}} = \frac{1}{2}\left(\mathbf{I}^{(a)}\right)^{\mathrm{H}}\mathbf{R}\mathbf{I}^{(b)}$$



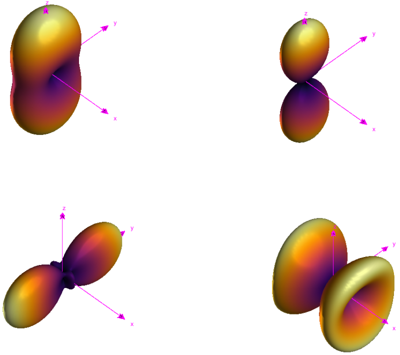
Radiated power.

Example – Radiated Powers

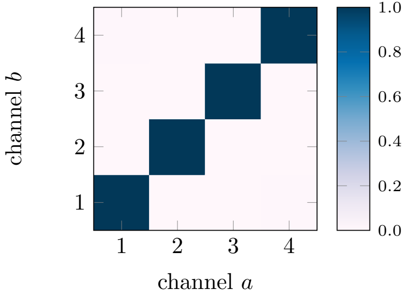


Radiated power of each channel for different initial vectors $\mathbf{v}$.

Example – Radiated Patterns



Normalized radiation patters of each channel.



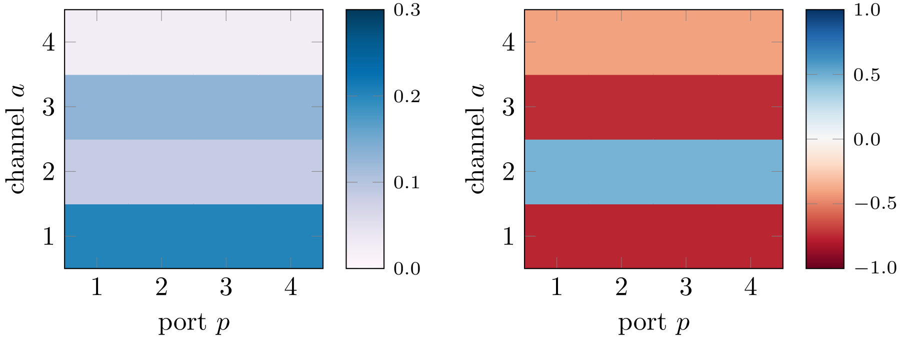
Envelope correlation coefficients[6].

[6] R. G. Vaughan and J. B. Andersen, "Antenna diversity in mobile communications," *IEEE Transactions on Vehicular Technology*, vol. 36, no. 4, pp. 149–172, 1987, ISSN: 0018-9545. DOI: 10.1109/T-VT.1987.24115

Example – Port Impedance

- Port impedance

$$\mathbf{Z}_p^{(a)} = \frac{V_p^{(a)}}{I_p^{(a)}}$$



Real and imaginary part of port impedance.

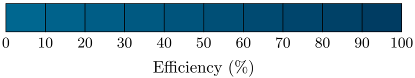
Conclusion

- Symmetries utilized for MIMO systems.
- Block-diagonalization of an arbitrary operator.
- Point group theory predicts:
  - Maximum number M of reachable uncorrelated channels,
  - number of delta gaps N needed to excite these channels.

Ongoing research

- Optimization of position.
- Capacity of channels.

n	2	3	4	5	6	7	8
C_n	2/2	3/3	4/4	5/5	6/6	7/7	8/8
D_n	4/4	4/6	6/8	6/10	8/12	8/14	10/16
C_{nv}	4/4	4/6	6/8	6/10	8/12	8/14	10/16
C_{nh}	4/4	6/6	8/8	10/10	12/12		
D_{nh}	8/8	8/12	12/16	12/20	16/24	16/28	20/32
D_{nd}	6/8	8/12	10/16	12/20	14/24	16/28	18/32
S_n	2/2		4/4		6/6		8/8
	T	T_h	T_d	O	T_h	I	I_h
	6/12	12/24	10/24	10/24	20/48	16/60	32/120



Ratio M/N for each point group.

Questions?

Michal Mašek
michal.masek@fel.cvut.cz

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version 1.0
The presentation is downloadable at
▶ michal-masek.cz

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