Amorphous Metamaterial with Negative Permeability

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***Abstract*— This paper investigates a metamaterial composed of ring resonators that are randomly distributed in space both in their positions and their orientation. Such metamaterial behaves as amorphous material with isotropic response. The main problem of the fabrication of this material is the spread of resonant frequencies of particular planar ring resonators. This spread is caused by tolerances of the fabrication process of planar resonators. Simulation shows that there is a limit of the dispersion of resonant frequencies allowing the metamaterial to behave as metamaterial with negative effective permeability. The amorphous metamaterial with negative permeability was fabricated and measurement verified the simulation.**

***Index Terms* — Metamaterial, amorphous structure, negative permeability, isotropic response, split ring resonator.**

I. INTRODUCTION

METAMATERIALS used in the microwave frequencies are designed as 3D periodic systems of resonant elements showing negative electric and/or magnetic polarizabilities. These resonators have mostly a planar structure. This 3D periodic system is an analogy of a crystalline material. In the view of this, amorphous material is represented by the system of resonant particles located in the 3D periodical net with positions randomly spread around the net nodes in all three directions with the spread equal to a half of period. Orientations of particles are randomly spread as well. This system can be used as metamaterial with an isotropic response, as it has been experimentally shown in [1,2] for metamaterials with negative permeability, and permittivity, respectively.

A 3D isotropic left-handed metamaterial based on the rotated transmission-line matrix (TLM) scheme was presented in [3] represented by a simplified planarized implementation. Similarly metamaterial with isotropic response composed of transmission lines was designed in [4]. Authors of [5] showed that the isotropic response of a 3D system of the planar resonant particles located on faces of cubes can be obtained by locating resonators with a proper symmetry. 3D metamaterial of bulk structure consisting of ring resonators and thin wires designed in [6] was proved to have almost nearly isotropic response. Novel fabrication technique of an isotropic IR metamaterial consisting of fourfold-symmetric 3D SRRs was proposed in [7]. Authors of [8] used a sample of randomly shaped and oriented SiC microparticles, and showed that such dielectric particles might be a much simpler alternative to SRRs for metamaterial fabrication in THz frequency band.

The fabrication preciseness of metamaterials is based namely on the fabrication of particular planar resonators. The material and geometric parameters of the resonators determine their resonance frequency, so variations of these parameters can be treated by accounting resonators with dispersed resonance frequencies [9]. Resonance frequency dispersion and random distribution of the positions result in widening the metamaterial response and reducing the values of its effective parameters. The band of negative effective permittivity and/or permeability can even disappear.

This paper, based on a preliminary works [10-13], studies the behavior the system of broad side coupled split ring resonators (BC-SRRs) presented in [1]. The simulation is verified by measuring transmission of the systems of BC-SRRs inserted in waveguide. The aim of this study is to define parameters of BC-SRRs to design the amorphous metamaterial showing negative permeability with isotropic response.

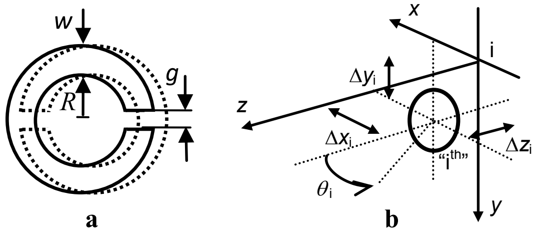

Fig. 1. The sketch of the layout of the planar broad-side coupled split ring resonator [1] (a). Position of the i^th resonant particle (loop) at the randomly chosen shifts in direction of axes Δx_i, Δy_i, and Δz_i corresponding to the lattice node with number i and with randomly chosen orientation defined by angle θ_i (b).

II. ANALYSIS - HOMOGENIZATION

The homogenization procedure inspired by Tretyakov [14] was applied to a system composed of BC-SRRs, Fig. 1a, located in a 3D periodic lattice with period a, that is assumed to be much smaller than the free space wavelength. The resonators have resonant frequencies spread according to normal distribution defined by central frequency and standard deviation σ. Positions of resonators are randomly spread within defined intervals Δx, Δy, Δz relating to the corresponding nodes of the lattice. Orientations of resonators are randomly spread in defined interval $\Delta\theta$ of angles relative to the z axis, see Fig. 1b. The orientation of the dipoles in the

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plane parallel to the xy plane is not accounted as the response does not depend on it. Resonators are modeled by point magnetic dipoles represented by small current circular loops.

The total averaged magnetic field that excites the structure is assumed to be aligned in the z direction $\hat{\mathbf{H}} = \hat{H}\mathbf{z}_o$. The local magnetic field exciting the reference magnetic dipole located at the origin of the coordination system can therefore be written as

$$\mathbf{H}_0 = \hat{\mathbf{H}} + \sum_{i\neq 0}\left(\mathbf{H}_i - \hat{\mathbf{H}}_i\right) - \hat{\mathbf{H}}_0 \quad . \tag{1}$$

Averaging is taken over the cell volume $V = a^3$. Field $\mathbf{H}_i$ is the field at the position of the reference dipole excited by a magnetic dipole located at position i. This field can be expressed by Eq. (9.35), p. 413 in [15]. In (1) the fields are summed over all magnetic dipoles, with the exception of $i = 0$, as the dipole cannot excite itself. The average value of the field excited by the reference dipole with $i = 0$ is [15]

$$\hat{\mathbf{H}}_0 = \frac{2}{3a^3}\mathbf{m}_0 , \tag{2}$$

which is exactly valid for a stationary field, but is acceptable here at the low frequency limit. Local magnetic fields $\mathbf{H}_i$ determine the magnetic moments of magnetic dipoles located generally in directions defined by unit vectors $\mathbf{m}_{oi}$

$$\mathbf{m}_i = \alpha_i(\omega, \omega_{oi})(\mathbf{H}_i \cdot \mathbf{m}_{oi})\mathbf{m}_{oi} \ , \tag{3}$$

where $\alpha(\omega, \omega_{oi})$ is resonator polarizability at frequency ω, ω_{0i} is resonance frequency. Equations (1) with the use of (2) and (3) represent the system of algebraic equations for unknown components of vectors $\mathbf{H}_i$. The effective permeability is now determined from the relation

$$\mathbf{B} = \mu_o\left(\hat{\mathbf{H}} + \mathbf{M}\right) = \mu_o\left(\hat{\mathbf{H}} + \frac{\mathbf{m}_0}{a^3}\right) , \tag{4}$$

inserting for the magnetic moment of the reference magnetic dipole (2) one gets assuming $\hat{H} = 1$ as a source quantity

$$\mu_{zz}(\omega, \omega_o) = 1 + \frac{\alpha_0(\omega, \omega_{o0})(\mathbf{H}_0 \cdot \mathbf{m}_{o0})\, m_{o0z}}{a^3} , \tag{5}$$

where m_{o0z} is the z component of unit vector $\mathbf{m}_{o0}$. Finally, the effective permeability is determined by integrating (5) multiplied by distribution function over resonance frequencies.

In the summation of (1), the resonance frequencies are randomly chosen around the central value, resonator positions are randomly chosen in respective to particular nodal points within defined intervals Δx, Δy, Δz, and orientations of resonators are randomly spread in defined interval $\Delta\theta$ of angles relative to the z axis, independently for each magnetic dipole. Finally (5) is averaged over a number of realizations chosen high enough to get the convergence. This averaging replaces N dimensional integration.

III. SIMULATION AND EXPERIMENT

The used BC-SRR investigated in [1], see Fig. 1a, were fabricated on a Rogers RT Duroid 5880 substrate 0.127 mm in thickness with permittivity 2.2 and 0.017 mm copper cladding. Dimensions marked in Fig. 1a are: $R = 1.8$ mm, $w = 0.7$ mm, $g = 0.3$ mm. Measurement done in [10] showed that the fabricated BC-SRR have normal distribution of resonance frequencies with standard deviation 25.2 MHz and center value 3.06 GHz. The period of the system is $a = 11$ mm.

The amorphous material is represented by a system of BC-SRRs located in the 3D periodical net with the period a. Their positions are randomly spread around the net nodes in all three directions by ±a/2 with the uniform distribution of probability. The orientations of the resonators are randomly chosen with the uniform distribution of probability within position angle θ measured in the respect of the z axis in interval between ±90 deg. The randomness is defined by the command RAND. These conditions correspond to the real metamaterial implementation with no preference of any position and any orientation in its realization, see Fig. 7.

The system of BC-SRRs was analyzed to determine the influence of particular quantities on the effective permeability to show the behavior of the amorphous metamaterial, and the condition to get negative permeability.

The strongest influence to the metamaterial behavior is caused by the dispersion of particular resonators resonance frequencies. Fig. 2 presents the effective permeability calculated for the standard deviation of the resonance frequencies dispersion varying from 0 to 60 MHz. For the standard deviation greater than 35 MHz the frequency band of negative permeability disappears.

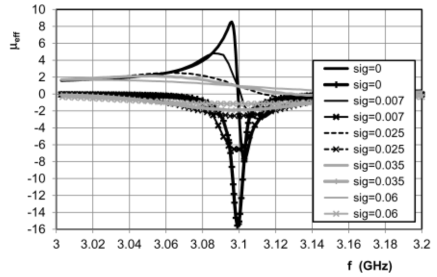

Fig. 2 Calculated effective permeability of the regular 3D periodic system of BC-SRRs depending on the dispersion of resonance frequencies with given standard deviation marked as “sig” and shown in GHz [10-12].

Measurement done in [10-12] confirmed result of the presented analysis of the system of BC-SRRs placed in the 3D periodic system all aligned in the direction of z-axis. Consequently this system represents the metamaterial with negative real part of effective permeability in the frequency band 3.15 up to 3.18 GHz with minimal value equal to -0.7 assuming $\sigma = 25$ MHz. Lower values, up to -8, can be obtained by reducing the standard deviation.

The other strong influence to the effective permeability is caused by random dispersion of the resonators orientation. The system of resonators without the dispersion of resonance frequencies shows for all widths of the interval $\Delta\theta$ from 0 till 180 deg the presence of the negative effective permeability as shown in Fig. 3. Fig. 4 shows the influence of the resonance frequencies dispersion. Data here are calculated for standard deviation of the resonance frequencies distribution equal to 0.025 GHz. Negative effective permeability disappears at interval of resonator particles positions higher than 120 deg.

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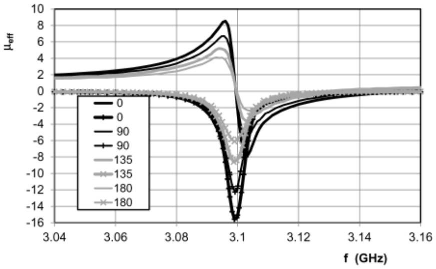

Fig. 3 Calculated effective permeability of the regular 3D periodic system of BC-SRRs depending on the interval $\Delta\theta$ within which resonator position angles are randomly selected. Resonance frequencies of particular resonators are assumed without dispersion.

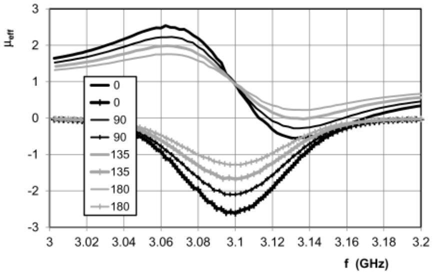

Fig. 4 The same dependence of the effective permeability as in Fig. 3 but calculated for standard deviation of resonance frequencies equal to 25 MHz.

BC-SRRs positions relative to the corresponding nodes of the 3D lattice represent last parameters taken into account. Fig. 5 shows results of the analysis with increasing spread of positions from zero till a. The influence of the randomly changed positions to the effective permeability dispersion is masked by the influence of the resonant frequency dispersion. The dependence shown in Fig. 5 is apparent only for very small values of the standard deviation of the resonance frequencies dispersion, taken there zero. All resonators are aligned in the z axis direction.

Finally the dependence of the effective permeability on the dispersion of resonators resonance frequencies is shown in Fig. 6. Data are calculated for the amorphous material defined above with fully random positions and orientations of all resonant particles in the system. The absolute value of effective permeability decreases with increasing standard deviation. Data in Fig. 6 show that the analyzed system behaves as metamaterial with negative effective permeability at standard deviation lower than 15 MHz. This is the main requirement for the fabrication process of the planar resonant particles of which the metamaterial is composed.

IV. AMORPHOUS METAMATERIAL WITH NEGATIVE PERMEABILITY

The amorphous metamaterial was assembled by planar resonators put into plastic shells in the form of a sphere of outer diameter 11 mm, see the inset of Fig. 7 [1]. Now the resonators are fully randomly distributed and oriented. The system consisting of 264 BC-SRRs in shells fills the cube with side length 72 mm, see Fig. 7. This cube was inserted into the raised R32 (WG10) waveguide and scattering parameters were measured. The response can be well compared here with data obtained by simulation done in CST Microwave Studio of the homogeneous block of metamaterial with effective permeability taken from Fig. 6 for standard deviation of resonance frequencies equal to 25 MHz. As Fig. 6 shows, this material does not exhibit negative permeability. That is due to too strong dispersion of resonance frequencies of BC-SRRs used in this composite metamaterial.

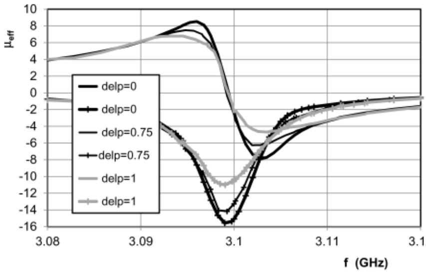

Fig. 5 Calculated effective permeability of the 3D system of BC-SRRs depending on the intervals $\Delta x = \Delta y = \Delta z$, marked by relative parameter $delp = \Delta x/a$, within which resonator positions are randomly spread around nodes of the 3D periodic net. All resonators are aligned in the z direction. Resonance frequencies are without dispersion.

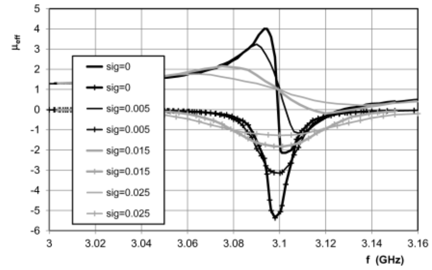

Fig. 6 Calculated effective permeability of the amorphous metamaterial depending on standard deviation of the resonance frequency distribution marked as "sig" and shown in GHz.

Fig. 7 The amorphous metamaterial in the partly disassembled measurement setup [1]. The inset shows the two halves of the shell with BC-SRR.

The presented results show that the BC-SRRs fabricated by standard printed circuit board technology on commercial substrate with standard deviation of resonance frequencies equal to 25 MHz cannot be used to create the amorphous metamaterial with negative effective permeability. The tolerance of the used substrate thickness is declared by the producer to be 10%, tolerance in permittivity about 1%. This cannot guarantee better results than standard deviation 25 MHz defined above. This problem has been solved by selecting by measurement the system of BC-SRRs with standard deviation equal to 5 MHz. This selected group of resonators does not guarantee exactly the normal distribution. The value of standard deviation is only a rough estimate.

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Transmission parameters of the block of these selected BC SRR measured in the raised R32 waveguide are compared in Fig. 8 with results of simulations done in the CST Microwave Studio of the metamaterial block of the same dimensions 72×72×54 mm. The effective permeability is taken from Fig. 6 for the standard deviation of resonance frequencies equal to 5 MHz. As it is evident from Fig. 6 this metamaterial has negative real part of effective permeability in frequency band from 3.1 up to 3.13 GHz with minimal value -1.5 at frequency 3.107 GHz. The agreement of data in Fig. 8 is relatively good. The problem is in approximation of the system of particular resonators by a continuum. That is because the fabricated resonators of radius 5 mm are not negligibly small comparing to the cell dimensions that are equal to the system period 11 mm, and there is a limited volume in the waveguide filled by the finite amount of resonators located in shells. Additionally, the used homogenization procedure models the resonators as point magnetic dipoles – small current loops so this also reduces the preciseness of the results as this condition is not exactly kept.

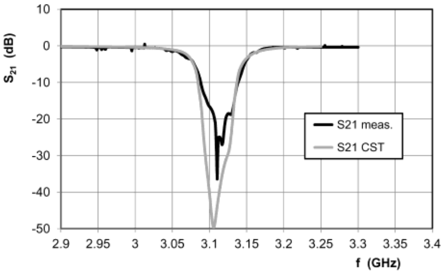

Fig. 8 Measured and calculated scattering parameters of the metamaterial block of selected BC-SRRs composing the amorphous metamaterial.

V. CONCLUSIONS

The random distribution of the resonance frequencies of particular resonators – BC-SRRs – constituting a metamaterial significantly widens the resonance frequency band of the effective permeability. The magnitude of the real part of the effective permeability decreases, the frequency band of its negative value is narrowed, and at higher values of the standard deviation of the resonance frequencies distribution this band even disappears. The spread of resonant particles in space represented by the spread of their positions relative to the 3D periodic net nodes, and by the spread of their orientations has similar influence to the metamaterial response. The spread of resonance frequencies plays however a dominant role, if all three sources of randomness are combined. The main requirement follows from this. The spread of resonance frequencies must be kept below a certain level to create the metamaterial exhibiting negative real part of effective permeability. The fabrication process of used planar resonant particles must be set correspondingly.

The amorphous metamaterial with negative real part of the effective permeability was designed, analyzed and fabricated. This metamaterial composed by a system of BC-SRRs located in plastic shells is ready to be applied as metamaterial with isotropic response. Its fabrication is very simple. The required volume is just filled by the plastic spherical shells containing resonant particles. That volume must have dimensions much higher than the shell diameter as the particles system could be assumed as a continuous system, and particle dimensions must be smaller than wavelength.

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