Accurate Evaluation of Characteristic Modes

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This talk concerns:

- electric currents in vacuum (generalization is, however, straightforward),
- time-harmonic quantities, \( i.e., \mathcal{A}(r,t) = \text{Re} \{ \mathcal{A}(r) \exp(j\omega t) \} \).
Characteristic Mode Decomposition

Generalized eigenvalue problem\(^1\)

\[ \mathbf{XI}_n = \lambda_n \mathbf{RI}_n, \]

\[ \mathbf{Z} = \mathbf{R} + j\mathbf{X} \in \mathbb{C}^{N \times N} \text{ is impedance matrix, } \mathbf{I}_n \in \mathbb{R}^{N \times 1} \text{ are expansion coefficients.} \]

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Benefits

- provide physical insight
- formalization of what antenna designers know and understand
- excellent entire-domain basis

Characteristic Mode Decomposition

Generalized eigenvalue problem\(^1\)

\[
X I_n = \lambda_n RI_n,
\]

\(Z = R + jX \in \mathbb{C}^{N \times N}\) is impedance matrix, \(I_n \in \mathbb{R}^{N \times 1}\) are expansion coefficients.

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but...
- hyped and sometimes misused (since used for everything)
- suffers from numerical problems
- incompatible with realistic feeding

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Benchmark of CM Solvers: \textbf{Spherical Shell}^2, \( ka = 1/2 \)

\[ \log_{10} |\lambda_n| \]

\text{TM/TE mode order}

\text{TM modes} \hspace{1cm} \text{TE modes}

\begin{tabular}{l}
\text{TM modes} \\
\text{TE modes} \\
\text{exact} \hspace{0.5cm} \text{AToM (1)} \\
\text{FEKO} \hspace{0.5cm} \text{AToM (8)} \\
\text{KS} \hspace{0.5cm} \text{WIPL-D} \\
\text{IDA} \hspace{0.5cm} \text{CEM One} \\
\text{CMC} \hspace{0.5cm} \text{Makarov} \\
\end{tabular}

Previous benchmark generated some important questions:

- How many modes can, in principle, be found?
- Is there a way how to increase their number?
- Is there a way how to accelerate solution if only few modes are needed?
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- Is there a way how to increase their number?
- Is there a way how to accelerate solution if only few modes are needed?

Problem is predominantly caused by numerical dynamics of the $\mathbf{R}$ matrix (naive interpretation: only a few modes radiate well. You will see later...).
Electric Field Integral Equation (EFIE)

EFIE for PEC bodies as the core of underlying MoM formulation:

\[
\hat{n} \times \mathbf{E}(r_2) = jk Z_0 \hat{n} \times \oint_{\Omega} \mathbf{G}(r_1, r_2) \cdot \mathbf{J}(r_1) \, dS_1,
\]

with dyadic Green function defined as

\[
\mathbf{G}(r_1, r_2) = \left(1 + \frac{1}{k^2} \nabla \nabla \right) \frac{e^{-jk|\mathbf{r}_1 - \mathbf{r}_2|}}{4\pi |\mathbf{r}_1 - \mathbf{r}_2|}.
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The impedance matrix \( \mathbf{Z} \) reads

\[
Z_{pq} = jk Z_0 \int_{\Omega} \int_{\Omega} \psi_p(\mathbf{r}_1) \cdot \mathbf{G}(\mathbf{r}_1, \mathbf{r}_2) \cdot \psi_q(\mathbf{r}_2) \, dS_1 \, dS_2.
\]
Spherical Wave Expansion of Dyadic Green Function

Spherical wave expansion of dyadic Green function reads\(^3\)

\[
G(r_1, r_2) = -jk \sum_{\alpha} u^{(1)}_{\alpha}(kr_<) u^{(4)}_{\alpha}(kr_>).
\] (4)

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\]

Impedance matrix \(Z\) with spherical wave expansion substituted

\[
Z_{pq} = k^2 Z_0 \sum_{\alpha} \int_{\Omega} \int_{\Omega} \psi_p(r_1) \cdot u^{(1)}_{\alpha}(kr_<) u^{(4)}_{\alpha}(kr>) \cdot \psi_q(r_2) \, dS_1 \, dS_2.
\tag{5}
\]

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(5)

can be used for reformulation of matrix \( R \) since \( u^{(1)}_{\alpha}(kr) = \text{Re}\{u^{(4)}_{\alpha}(kr)\} \) as

\[ R_{pq} = k^2 Z_0 \sum_{\alpha} \int_{\Omega} \psi_p(r_1) \cdot u^{(1)}_{\alpha}(kr_1) \, dS_1 \int_{\Omega} u^{(1)}_{\alpha}(kr_2) \cdot \psi_q(r_2) \, dS_2 \]  

(6)

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\[ \text{G. Kristensson, Scattering of electromagnetic waves by obstacles. Edison, NJ: SciTech Publishing, an imprint of the IET, 2016} \]
Definition of Projection Matrix $\mathbf{S}$

Resistance matrix $\mathbf{R}$ is expressed as a product of two identical rectangular matrices:

$$ R_{pq} = \sum_{\alpha} \left( k \sqrt{Z_0} \int_{\Omega} \psi_p(r_1) \cdot \mathbf{u}_\alpha^{(1)}(kr_1) \, dS_1 \right) \left( k \sqrt{Z_0} \int_{\Omega} \mathbf{u}_\alpha^{(1)}(kr_2) \cdot \psi_q(r_2) \, dS_2 \right) $$
Definition of Matrix $S$

Resistance matrix $R$ is expressed as a product of two identical rectangular matrices:

\[
R_{pq} = \sum_{\alpha} \left( k \sqrt{Z_0} \int_{\Omega} \psi_p(r_1) \cdot u^{(1)}_{\alpha}(k r_1) \, dS_1 \right) \left( k \sqrt{Z_0} \int_{\Omega} u^{(1)}_{\alpha}(k r_2) \cdot \psi_q(r_2) \, dS_2 \right)
\]

Definition\(^4\) of the matrix $S \in \mathbb{R}^{N_{\alpha} \times N}$

\[
S_{\alpha p} = k \sqrt{Z_0} \int_{\Omega} \psi_p(r) \cdot u^{(1)}_{\alpha}(k r) \, dS,
\]

and its relation to the resistance matrix

\[
R = S^T S.
\]

---

Matrix $S$ is real-valued, rectangular, low-rank $N_\alpha = 2L(L+2)$.

$$L = \left\lceil k_\alpha + \frac{7}{3} \sqrt{k_\alpha + 2} \right\rceil.$$  

Matrix $S^T S$ does not contain any negative eigenvalue higher than numerical noise.

Matrix $S$ represents projection between RWGs and spherical waves, i.e.,

$$R = S^T S,$$

$$R_{\text{sph}} = SS^T.$$  

$\alpha$: spherical waves

$N_\alpha = 510, N = 900$

$N_\alpha = 510, N = 721$

Double precision
Properties of Matrix $\mathbf{S}$, Part #1

- Matrix $\mathbf{S}$ is real-valued, rectangular, low-rank

$$N_\alpha = 2L(L + 2), \quad (8)$$
$$L = \lceil ka + 7\sqrt[3]{ka} + 2 \rceil. \quad (9)$$

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$$\mathbf{R} = \mathbf{S}_T \mathbf{S}, \quad (10)$$
$$\mathbf{R}_{sph} = \mathbf{S}_S \mathbf{S}_T. \quad (11)$$


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Spherical shell, $N_\alpha = 510, N = 900$

Rectangular plate, $N_\alpha = 510, N = 721$
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\[
\sqrt{S_\alpha S_\alpha^T}
\]

\[\alpha: \text{spherical waves}\]

\[\alpha: \text{double precision}\]

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Radiated power can be calculated as

\[ P_{\text{rad}} = \frac{1}{2Z_0} \int_{S^2} |F(\hat{r})|^2 \ dS \approx \frac{1}{2} I^H R I = \frac{1}{2} |S I|^2 = \frac{1}{2} \sum_{\alpha} |f_{\alpha}|^2 \]  \hspace{1cm} (12)

with

\[ F(\hat{r}) = \frac{1}{k} \sum_{\alpha} j^{l-\tau+2} f_{\alpha} Y_{\alpha}(\hat{r}), \]  \hspace{1cm} (13)

where \( Y_{\alpha}(\hat{r}) \) are the real-valued spherical vector harmonics.
Properties of Matrix $\mathbf{S}$, Part #2

Radiated power can be calculated as

$$P_{\text{rad}} = \frac{1}{2Z_0} \int_{S^2} |\mathbf{F}(\hat{r})|^2 \, dS \approx \frac{1}{2} \mathbf{I}^H \mathbf{R} \mathbf{I} = \frac{1}{2} |\mathbf{S}\mathbf{I}|^2 = \frac{1}{2} \sum_{\alpha} |f_\alpha|^2$$  \hspace{1cm} (12)

with

$$\mathbf{F}(\hat{r}) = \frac{1}{k} \sum_{\alpha} j^{l-\tau+2} f_\alpha \mathbf{Y}_\alpha(\hat{r}),$$  \hspace{1cm} (13)

where $\mathbf{Y}_\alpha(\hat{r})$ are the real-valued spherical vector harmonics.

<table>
<thead>
<tr>
<th>Example</th>
<th>$N_\alpha$</th>
<th>$N$</th>
<th>Comp. times in IDA (s)</th>
<th>$\mathbf{R}$</th>
<th>$\mathbf{S}$</th>
<th>$\mathbf{R} = \mathbf{S}^T \mathbf{S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>spherical shell</td>
<td>880</td>
<td>750</td>
<td></td>
<td>0.09</td>
<td>0.009</td>
<td>0.011</td>
</tr>
<tr>
<td>spherical shell</td>
<td>880</td>
<td>3330</td>
<td></td>
<td>1.78</td>
<td>0.039</td>
<td>0.083</td>
</tr>
<tr>
<td>helicopter</td>
<td>880</td>
<td>18898</td>
<td></td>
<td>54.50</td>
<td>0.236</td>
<td>1.660</td>
</tr>
</tbody>
</table>
Singular value decomposition (SVD) of matrix $S$

$$S = UV^H,$$  \hspace{1cm} (14)

substituted into CM definition gives

$$(V^H XV)(V^HI_n) = \lambda_n (\Lambda^H \Lambda) (V^HI_n) \longrightarrow \tilde{X}I_n = \lambda_n \tilde{R}I_n,$$  \hspace{1cm} (15)
CMs Using SVD of matrix $S$ and GEP Partitioning

Singular value decomposition (SVD) of matrix $S$

$$S = U\Lambda V^H,$$  \hspace{1cm} (14)

substituted into CM definition gives

$$\begin{pmatrix} V^HXV \end{pmatrix} \begin{pmatrix} V^HI_n \end{pmatrix} = \lambda_n \begin{pmatrix} \Lambda^H \Lambda \end{pmatrix} \begin{pmatrix} V^HI_n \end{pmatrix} \quad \rightarrow \quad \tilde{X}I_n = \lambda_n \tilde{R}I_n,$$ \hspace{1cm} (15)

Partitioning

$$\tilde{X}I = \begin{pmatrix} \tilde{X}_{11} & \tilde{X}_{12} \\ \tilde{X}_{21} & \tilde{X}_{22} \end{pmatrix} \begin{pmatrix} \tilde{I}_{1n} \\ \tilde{I}_{2n} \end{pmatrix} = \begin{pmatrix} \lambda_{1n} \tilde{R}_{11} \tilde{I}_{1n} \\ 0 \end{pmatrix}$$ \hspace{1cm} (16)

and reducing to Schur complement yields the final GEP formulation

$$\begin{pmatrix} \tilde{X}_{11} - \tilde{X}_{12} \tilde{X}_{22}^{-1} \tilde{X}_{21} \end{pmatrix} \tilde{I}_{1n} = \lambda_{1n} \tilde{R}_{11} \tilde{I}_{1n}.$$ \hspace{1cm} (17)
Modification of Generalized Eigenvalue Problem

Properties of the Decomposition:

Characteristic modes are constructed as

\[ \tilde{I}_n = \begin{pmatrix} \tilde{I}_{1n} \\ -\tilde{X}^{-1}_{22} \tilde{X}_{21} \tilde{I}_{1n} \end{pmatrix}, \]  

(18)

radiated power is implicitly normalized by \( \Lambda^H \Lambda \) matrix in (15)

\[ \tilde{I}_n^H \tilde{R} \tilde{I}_m = \delta_{nm}. \]

(19)

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Properties\(^4\):

- numerical dynamics doubled thanks to the SVD and partitioning,
- number of used spherical modes controls the number of CMs,
- for \( N_{\alpha} \ll N \) (always fulfilled in ESA regime) remarkable speed-up.

Spherical Shell

Decomposition With Matrix $S$

| TM/TE mode order | $\log_{10} |\lambda_n|$ |
|------------------|------------------|
| 168              | 10               |
| 120              | 10               |
| 80               | 10               |
| 48               | 10               |
| 24               | 10               |
| 8                | 10               |

- **TM modes**
- **TE modes**

- **exact**
- **$R, X, FEKO$**
- **$R, X, AToM$**

Spherical Shell

![Graph showing TM/TE mode order vs. log₁₀ |λₙ| for TM and TE modes, with data points for exact, R, X, FEKO, and R, X, AToM comparisons]
Spherical Shell

Decomposition With Matrix $S$

100 modes were calculated (eigs)

- \((X, R)\) 0.7 s (29)
- \((X, R) + \text{Advanpix}: 1324\) s
- \((\tilde{X}, \tilde{R})\) 0.5 s (37)

(If matrix \(S\) is reduced, calculation further accelerated.)
Decomposition With Matrix $S$

Rectangular Plate

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Two high-order modes of rectangular plate:

- left: inductive, $n = 17$, 
  $\lambda_{17} = 2.461 \cdot 10^{17}$,
- right: capacitive, $n = 77$, 
  $\lambda_{77} = -1.947 \cdot 10^{24}$. 
Two high-order modes of rectangular plate:

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Such high-order modes are not needed in practice (except tracking).

- However, accuracy can be interchanged for comp. speed.
Acceleration of the CMs Decomposition

If double precision is enough, however, computational speed is required:

\[ \mathbf{X}_I n = \lambda_n \mathbf{S}^T \mathbf{S} I_n \]  \hspace{1cm} (20)

Properties:
- Solved in basis of spherical waves (\( \hat{I}_n = \mathbf{S} I_n \)),
- Standard (not generalized) eigenvalue problem,
- Solution of typically small \( N \alpha \times N \alpha \) eigenvalue problem (extreme speed-up),
- All modes, well-represented in the spherical basis, are found,
- eig shall be used instead of eigs in MATLAB.
Acceleration of the CMs Decomposition

If double precision is enough, however, computational speed is required:

\[ \mathbf{XI}_n = \lambda_n \mathbf{S}^T \mathbf{SI}_n \]  (20)

\[ \mathbf{SI}_n = \lambda_n \mathbf{SX}^{-1} \mathbf{S}^T \mathbf{SI}_n \quad \rightarrow \quad \hat{\mathbf{XI}}_n = \xi_n \hat{\mathbf{I}}_n \]  (21)

with \( \hat{\mathbf{X}} = \mathbf{SX}^{-1} \mathbf{S}^T \), \( \hat{\mathbf{I}}_n = \mathbf{SI} \), and \( \xi_n = 1/\lambda_n \).
Decomposition With Matrix $S$

**Acceleration of the CMs Decomposition**

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$$\mathbf{XI}_n = \lambda_n \mathbf{S}^T \mathbf{SI}_n \quad (20)$$

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**Properties:**

- solved in basis of spherical waves ($\hat{\mathbf{I}}_n = \mathbf{SI}$),
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## Acceleration of the CMs Decomposition – Comparison

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<th>($\mathbf{R}, \mathbf{X}$)</th>
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<th>($\mathbf{S}\mathbf{X}^{-1}\mathbf{S}^T$)</th>
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<tr>
<td>rectangular plate</td>
<td>100</td>
<td>510</td>
<td>655</td>
<td>0.7</td>
<td>0.8</td>
<td>0.5</td>
<td>(510 modes)</td>
</tr>
<tr>
<td>spherical shell</td>
<td>300</td>
<td>880</td>
<td>3330</td>
<td>29</td>
<td>6.7</td>
<td>2.6</td>
<td>(880 modes)</td>
</tr>
<tr>
<td>helicopter</td>
<td>25</td>
<td>880</td>
<td>18898</td>
<td>149</td>
<td>170</td>
<td>47</td>
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<td>helicopter</td>
<td>100</td>
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<td>18898</td>
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<td>173</td>
<td>47</td>
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Windows Server 2012, 2×Xeon E5-2665 @ 2.4 GHZ, 72 GB RAM
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Windows Server 2012, 2×XEON E5-2665 @ 2.4 GHZ, 72 GB RAM

- $(\tilde{R}, \tilde{X})$ gives *significantly more* modes accurately and is *typically faster*.
- $(SX^{-1}S^T)$ gives *slightly more* modes accurately and is *significantly faster*.
- $(SX^{-1}S^T)$ finds all modes available from a given set of spherical harmonics.
- $(SX^{-1}S^T)$ decomposition is excellent for high $ka$ with large DOFs $N$. 

Capek, M., et al.
Restriction to TM/TE Modes

Matrix $S^{TM/TE} = S(i,:)$ contains TE and TM modes in separate rows.
Restriction to TM/TE Modes

Matrix $S^{TM/TE} = S(i,:) \text{ contains TE and TM modes in separate rows.}$
Restriction to TM/TE Modes

Matrix \( S^{\text{TM/TE}} = S(i,:) \) contains TE and TM modes in separate rows.

![Graph showing TM/TE mode order and \( \log_{10} |\lambda_n| \) for different methods: exact, \( \tilde{R}, \tilde{X} \) (TE only), \( R, X, \text{AToM} \), \( \tilde{R}, \tilde{X} \) (TM only).]
Concluding Remarks

Conclusions

- New matrix operator based on MoM formalism,
- matrix $S$ has controllable and predictable behavior and numerically neat properties,
- matrix $S$ has many applications (some of them probably yet unknown),
- if $X$ is not needed, matrix $S$ should be preferred over $R$,
- with respect to the (characteristic) modes, the matrix $S$ is, in certain sense, a return to their scattering origin.$^5$

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Dominant characteristic mode of helicopter model discretized into 18989 RWGs, $ka = 1/2$. 
Questions?

For a complete PDF presentation see [capek.elmag.org](capek.elmag.org)

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