Excitation of Optimal and Suboptimal Currents

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This talk concerns:

- electric currents in vacuum,
- time-harmonic quantities, \( i.e., A(r, t) = \text{Re} \{ A(r) \exp(j\omega t) \} \).
Optimal Currents – What Are They?

A current $\mathbf{J} = \mathbf{J}(r, \omega)$, $r \in \Omega$, is denoted $\mathbf{J}_{\text{opt}}$ and called as optimal current\(^1\) if

$$\langle \mathbf{J}_{\text{opt}}, \mathbf{L}(\mathbf{J}_{\text{opt}}) \rangle = \min_J \langle \mathbf{J}, \{\mathbf{L}(\mathbf{J})\} \rangle = p_{\min},$$  \hspace{1cm} (1)

$$\langle \mathbf{J}_{\text{opt}}, \mathbf{M}_n(\mathbf{J}_{\text{opt}}) \rangle = q_n,$$  \hspace{1cm} (2)

$$\langle \mathbf{J}_{\text{opt}}, \mathbf{N}_n(\mathbf{J}_{\text{opt}}) \rangle \leq r_n.$$  \hspace{1cm} (3)

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Optimal Currents – What Are They?

A current \( J = J(r, \omega), r \in \Omega \), is denoted \( J_{\text{opt}} \) and called as optimal current\(^1\) if

\[
\langle J_{\text{opt}}, \mathcal{L}(J_{\text{opt}}) \rangle = \min_J \langle J, \{ \mathcal{L}(J) \} \rangle = p_{\text{min}},
\]

\[
\langle J_{\text{opt}}, \mathcal{M}_n(J_{\text{opt}}) \rangle = q_n,
\]

\[
\langle J_{\text{opt}}, \mathcal{N}_n(J_{\text{opt}}) \rangle \leq r_n.
\]

What are the optimal currents good for?

- They establish fundamental bounds of \( p = \langle J, \mathcal{L}(J) \rangle \) for a given \( \Omega \) and \( \omega \).

**Use case:** Minimum quality factor \( Q \) for electrically small antennas.

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Minimization of Quality Factor $Q$

Current $J_{\text{opt}}$ minimizing quality factor $Q$ of a given shape $\Omega$:

$$Q(J_{\text{opt}}) = \min_J \{Q(J)\}$$ (4)
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Rao-Wilton-Glisson basis functions

$$J(r) \approx \sum_n I_n \psi_n(r)$$  (5)

$$Q(I) = \frac{2\omega \max\{W_m, W_e\}}{P_r} = \max\left\{ \frac{I^H X_m I, I^H X_e I}{I^H R I} \right\}$$  (6)

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Minimum Quality Factor $Q$

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We know several efficient minimization procedures$^2$.

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Basis of Characteristic Modes

Diagonalization of impedance matrix $Z = R + jX$ as\(^3\)

$$XI_m = \lambda_m RI_m$$  \hspace{1cm} (7)

- useful set of entire-domain basis functions,

$$I = \sum_m \alpha_m I_m$$  \hspace{1cm} (8)

- only few modes needed to represent ESAs

$$\left(1 + j\lambda_m\right) \delta_{mn} = \frac{1}{2} I_m^H Z I_n.$$  \hspace{1cm} (9)

- meant originally for scattering problems\(^4\).

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Optimal current can be approximated\(^5\) by

\[
Q(I_{\text{opt}}) \approx Q(I_1 + \alpha_{\text{opt}}I_2)
\]

\[\alpha_{\text{opt}} = \sqrt{-\frac{\lambda_1}{\lambda_2}}e^{-j\varphi} = \sqrt{-\frac{I_1^T X I_1}{I_2^T X I_2}}e^{-j\varphi}, \quad \varphi \in [-\pi, \pi]\]

The optimization problem can be advantageously solved in other bases as well!
Modal Composition of the Optimal Current $J_{\text{opt}}$

Optimal current with respect to minimum quality factor $Q$. 
Solution Expressed in Characteristic Modes

Modal Composition of the Optimal Current $J_{\text{opt}}$

Optimal current with respect to minimum quality factor $Q$.

Dominant (dipole-like) characteristic mode $J_1$.

First inductive (loop-like) mode $J_2$, $\alpha_2 = 0.4553$. 
Alternative Bases

- Stored energy modes\textsuperscript{6}
  \[ \omega \frac{\partial \mathbf{X}}{\partial \omega} \mathbf{I}_m = q_m \mathbf{R} \mathbf{I}_m, \]  
  \[ (12) \]

- Minimum quality factor $Q$ modes\textsuperscript{7}
  \[ ((1 - \nu) \mathbf{X}_m + \nu \mathbf{X}_e) \mathbf{I}_m = Q_{\nu m} \mathbf{R} \mathbf{I}_m, \]  
  \[ (13) \]

- Optimal gain $G$ including losses in metalization\textsuperscript{8}
  \[ \mathbf{U} (\hat{e}, \hat{r}) \mathbf{I}_m = \zeta_m \frac{1}{8\pi} (\mathbf{R} + \mathbf{R}_\rho) \mathbf{I}_m, \]  
  \[ (14) \]

- Optimal radiation efficiency\textsuperscript{8}
  \[ \mathbf{R} \mathbf{I}_m = \zeta_m (\mathbf{R} + \mathbf{R}_\rho). \]  
  \[ (15) \]

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Excitation of Optimal Currents

Optimal current $I_{opt}$ for minimal quality factor $Q$.

- How to feed optimal currents?
Optimal current $I_{\text{opt}}$ for minimal quality factor $Q$. Feeding map (abs values) for optimal current $I_{\text{opt}}$.

How to feed optimal currents?

- $V_{\text{opt}} = ZI_{\text{opt}}$

$$I = \sum_{n} \frac{I_{n}^{H}V}{1 + j\lambda_{n}} \frac{I_{n}^{H}R_{n}I_{n}}{I_{n}^{H}R_{n}I_{n}}$$

(16)

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Excitation of Optimal Currents

Optimal current $I_{\text{opt}}$ for minimal quality factor $Q$.

Feeding map (abs values) for optimal current $I_{\text{opt}}$.

How to feed optimal currents?

1. $V_{\text{opt}} = ZI_{\text{opt}}$
   - Impressed currents in vacuum.
   - Shape has to be modified.
   - Can modal techniques help?

$$I = \sum_{n} \frac{I_n^H V}{1 + j\lambda_n} \frac{I_n}{I_n^H R I_n}$$  \hspace{1cm} (16)
How to Excite the Optimal Currents

Let us try to modify structure manually.

- A loop.
- 2 modes = at least two feeders?

Dependence of \( Q_{\text{min}} \) on number of (optimally placed) feeders.
How to Excite the Optimal Currents

Dependence of $Q_{\text{min}}$ on number of (optimally placed) feeders.

Let us try to modify structure manually.
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- Rectangle: $Q_{\text{min}} = 69.5$
- Loop: $Q_{\text{min}} = 78.9$

Excited Characteristic Modes

As expected, solution represented by two CMs.

Even to excite two CMs properly, many feeders needed.

Dependence of ME coef. $|\alpha_n|$ on number of (optimally placed) feeders.
Antenna synthesis – how far can we go?

- On the present, only the heuristic optimization\(^9\),
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- On the present, only the heuristic optimization\(^9\),
- triangles and edges can be subjects of pixelization.

Computational time: 12116 s

Result of heuristic structural optimization using MOGA NSGAI\(\text{II}\) from AToM-FOPS.
Antenna synthesis – how far can we go?

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\[ \frac{Q(I)}{Q(I_{opt})} = 1.811 \]

Result of heuristic structural optimization using MOGA NSGAI from AToM-FOPS.

Resulting sub-optimal current approaching minimal value of quality factor Q.
Complexity of the Problem

- shape modification resembles NP-hard problem
- any extra feeder levels up the complexity enormously
Structure of the Solution Space

### Complexity of the Problem

- shape modification resembles NP-hard problem
- any extra feeder levels up the complexity enormously

<table>
<thead>
<tr>
<th>How much DOF we have?</th>
<th>28</th>
<th>52</th>
<th>120</th>
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<td>$N$ (unknowns)</td>
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- possibilities
- unique solutions

Complexity of geometrical optimization for given voltage gap (red line) and $N$ unknowns.
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<tr>
<td>Possibilities</td>
<td>$5.24 \cdot 10^{29}$</td>
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<tr>
<td>Unique solutions</td>
<td>$2.68 \cdot 10^{8}$</td>
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Complexity of geometrical optimization for given voltage gap (red line) and $N$ unknowns.
Structure of Solution Space

- all combinations for $N = 28$ edges ($5.24 \cdot 10^{29}$) calculated in Matlab$^{10}$
  - 3 days on supercomputer, 2 resulting vectors + permutation table $\approx 55$ GB
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Structure of all suboptimal solution within 2\% tolerance to the best found candidate. Edge no. 18 is fed.
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Structure of all suboptimal solution within 2% tolerance to the best found candidate. Edge no. 18 is fed.

Number of solutions in dependence on their quality factor $Q$. The best solution reaches $Q(\Omega_{opt}) \approx 292$. 

Naive Alternative to Heuristic Algorithms

Deterministic algorithm dealing with shape optimization

- The worst edge (causing high quality factor $Q$) is iteratively removed.
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Computational time: 1155 s

Result of deterministic in-house algorithm removing in each iteration “the worst” edge.
Naive Alternative to Heuristic Algorithms

Deterministic algorithm dealing with shape optimization
▶ The worst edge (causing high quality factor $Q$) is iteratively removed.

Computational time: 1155 s

$Q(I)/Q(I_{opt}) = 1.813$

Result of deterministic in-house algorithm removing in each iteration “the worst” edge.

Resulting current given by in-house deterministic algorithm.
Mesh grid converted to graph.

- Longest cycle (loop) or path (dipole) in a mesh are NP hard.
- Can adaptive meshing help?
- Convergence of mesh grid has to be controlled.
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Mesh grid converted to graph.

Can we somehow combine heuristic and our knowledge?
Current and Antenna Optimization

Current optimization

▶ lower bounds,

Antenna optimization

▶ real performance,

Can modal techniques help?

▶ Understanding and interpretation of the solution.

▶ For matrix compression, i.e.,\[ A_{\text{red}} = [I_{\text{H}} A_{\text{full}} I_{\text{N}}]. \]
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Current and Antenna Optimization

Current optimization
- lower bounds,
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- convex optimization,

Antenna optimization
- real performance,
- NP-hard (NP-complete),
- heuristic optimization,

Can modal techniques help?
- Understanding and interpretation of the solution.
- For matrix compression, i.e., $A_{red} = \begin{bmatrix} I & H \end{bmatrix} A_{full} \begin{bmatrix} I \\ I \end{bmatrix}$.
- New operators $\rightarrow$ new decompositions.
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- lower bounds,
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- real performance,
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- lower bounds,
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- real performance,
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- \( n \ll N \) feeders.

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- New operators \( \rightarrow \) new decompositions.
Questions?

For a complete PDF presentation see [capek.elmag.org](capek.elmag.org)

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