Benchmarking of Characteristic Modes and Their (Selective) Excitation

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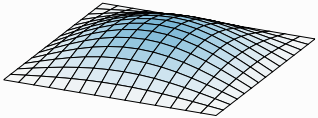
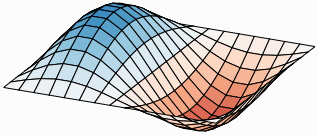
Outline

1. Benchmarking of Characteristic Modes
   - Why should we care?
   - Do we know results analytically?
2. Spherical Shell as a Testing Case
   - Short Historical Excursion
   - Analytically Known Results
   - Analytical $\times$ Numerical Results
3. Representation of Optimal Currents
4. Excitation of Characteristic Modes



$\boldsymbol{\mathcal{J}}_1(\boldsymbol{r}, t)$



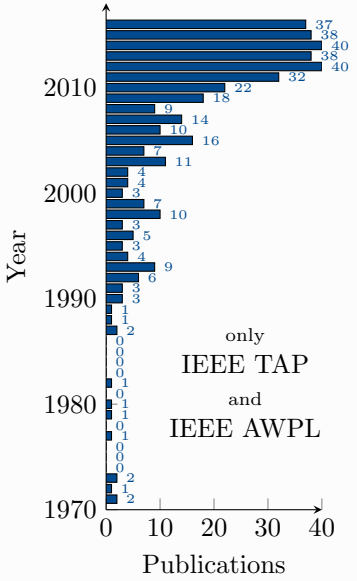
$\boldsymbol{\mathcal{J}}_2(\boldsymbol{r}, t)$

In this talk:

- Only PEC objects.

Why Benchmarking?

- (recent) boom of characteristic modes
- plenty of scientific data have been produced



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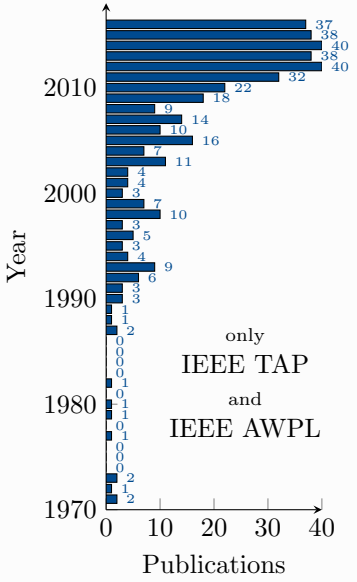
How precisely are our results calculated?



Why Benchmarking?

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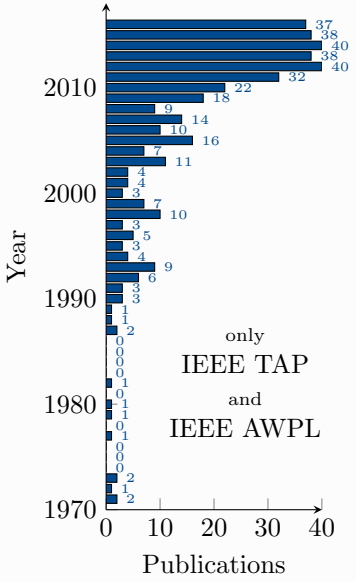
- plenty of scientific data have been produced

How precisely are our results calculated?

In this presentation[1]

- analytical solution,
- validation of solvers,
- investigation of convergence.



[1] Paper is prepared on this topic. All the data are subject of ongoing research.

Separable Systems

$$\nabla^2 f + k^2 f = 0$$

Analytical form of CMs potentially exists only in 6 coordinate systems[2] in which vector Helmholtz equations is separable[3]

[2] J. A. Stratton, *Electromagnetic theory*. Wiley – IEEE Press, 2007

[3] W. W. Hansen, "A new type of expansion in radiation problems", *Phys. Rev.*, vol. 47, no. 2, pp. 139–143, 1935. DOI: http://dx.doi.org/10.1103/PhysRev.47.139

Separable Systems

Analytical form of CMs potentially exists only in 6 coordinate systems[2] in which vector Helmholtz equations is separable[3]

- Li, Kang and Leong[4]: "So far, there are only six coordinate systems in which the scalar Helmholtz equations are separable and solenoidal solutions of the vector Helmholtz equations which are transverse to coordinate surfaces are obtainable."

$$\nabla^2 f + k^2 f = 0$$

$$\begin{aligned} \boldsymbol{L} &= \nabla f \\ \boldsymbol{M} &= \nabla \times \hat{\boldsymbol{a}} f \\ \boldsymbol{N} &= \frac{1}{k} \nabla \times \boldsymbol{M} \\ \boldsymbol{M} &= \boldsymbol{L} \times \hat{\boldsymbol{a}} \\ \boldsymbol{M} &= \frac{1}{k} \nabla \times \boldsymbol{N} \end{aligned}$$

$$\begin{aligned} \nabla \times \boldsymbol{L} &= 0 \\ \nabla \cdot \boldsymbol{M} &= 0 \\ \nabla \cdot \boldsymbol{N} &= 0 \end{aligned}$$

[2] J. A. Stratton, *Electromagnetic theory*. Wiley – IEEE Press, 2007

[3] W. W. Hansen, "A new type of expansion in radiation problems", *Phys. Rev.*, vol. 47, no. 2, pp. 139–143, 1935. DOI: http://dx.doi.org/10.1103/PhysRev.47.139

[4] L.-W. Li, X.-K. Kang, and M.-S. Leong, *Spheroidal wave functions in electromagnetic theory*. 2002

Solvable Systems

To get analytical results we should selected one of the following systems:

- rectangular
- spherical
- circular-cylinder
- elliptic-cylinder
- parabolic-cylinder
- conical

Solvable Systems

To get analytical results we should selected one of the following systems:

rectangular	of infinite extent in $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ (plane waves)
spherical	can be done
circular-cylinder	of infinite extent in $\hat{\mathbf{z}}$ (cylindrical waves)
elliptic-cylinder	of infinite extent in $\hat{\mathbf{z}}$ (cylindrical waves)
parabolic-cylinder	of infinite extent in $\hat{\mathbf{z}}$ (cylindrical waves)
conical	for heros only

For other systems, the separability of vector Helmholtz equations is problematic[5].

[5] E.g. for prolate and oblate spheroids, the vector wave functions are not orthogonal to radial spheroidal functions. This induces problems with solving the boundary value problems correctly. Discuss Li, Kang and Leong's book for details.

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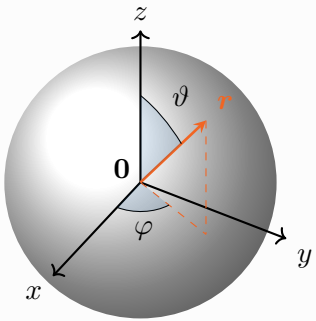
Characteristic Modes of Spherical Shell



Spherical shell of radius a.

Characteristic modes $\boldsymbol{J}_n$ are defined[6] here as

$$\int_{\Omega} \boldsymbol{J}_m^* \cdot \mathcal{Z}\left(\boldsymbol{J}_n\right) \mathrm{d}\Omega = \left(1 + \mathrm{j}\lambda_n\right) \delta_{mn}, \tag{1}$$

in which

$$\mathcal{Z}\left(\boldsymbol{J}_n\right) = \mathcal{R}\left(\boldsymbol{J}_n\right) + \mathrm{j}\mathcal{X}\left(\boldsymbol{J}_n\right) = -\hat{\boldsymbol{r}} \times \hat{\boldsymbol{r}} \times \boldsymbol{E}^{\mathrm{s}}\left(\boldsymbol{J}_n\right) \tag{2}$$

and

$$\boldsymbol{E}^{\mathrm{s}}\left(\boldsymbol{J}\right) = -\frac{\mathrm{j}Z_0}{4\pi} \int_{\Omega} \left(k\boldsymbol{J} + \frac{1}{k}\nabla\nabla \cdot \boldsymbol{J}\right) \frac{\mathrm{e}^{-\mathrm{j}kR}}{R} \mathrm{d}\Omega \tag{3}$$

[6] Definition is based on fundamental property we require from CMs, i.e., ability to diagonalize EFIE operator.

Short Comment on Defining GEP

Conventional approach via generalized eigenvalue problem, i.e.,

$$\mathcal{X}\left(\boldsymbol{J}_n\right) = \lambda_n \mathcal{R}\left(\boldsymbol{J}_n\right) \tag{4}$$

is an alternative here, however, explicitly

$$\int_{\Omega} \left(\boldsymbol{J}_n + \frac{\nabla_{\mathrm{t}} \nabla \cdot \boldsymbol{J}_n}{k^2} \right) \frac{\cos\left(kR\right)}{R} \,\mathrm{d}\Omega = \lambda_n \int_{\Omega} \left(\boldsymbol{J}_n + \frac{\nabla_{\mathrm{t}} \nabla \cdot \boldsymbol{J}_n}{k^2} \right) \frac{\sin\left(kR\right)}{R} \,\mathrm{d}\Omega \tag{5}$$

should be solved.

- Formula (5) suggests an answer to the questions of what far-field system should be used to preserve the orthogonality of modes.

How to Attack the Problem?

Remarkable milestones (and interesting reading)

Garbacz scattering CMs, spherical shell presented[7],

Harrington and Mautz first numerical implementation[8] (and benchmark!).

Antonino-Daviu CMs for separable systems[9]

[7] R. J. Garbacz, "A generalized expansion for radiated and scattered fields", PhD thesis, Department of Electrical Engineering, Ohio State University, 1968

[8] R. F. Harrington and J. R. Mautz, "Computation of characteristic modes for conducting bodies", *IEEE Trans. Antennas Propag.*, vol. 19, no. 5, pp. 629–639, 1971. DOI: 10.1109/TAP.1971.1139990

[9] E. Antonino-Daviu, "Analysis and design of antennas for wireless communications using modal methods", PhD thesis, UPV, 2008

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Initial guess of the proper orthogonal system has to be done. Rationale:

1. CMs are modes of **S** matrix (Garbacz),
2. spherical harmonics diagonalize **S** for spherical shell,
3. let us use spherical harmonics for diagonalization of **Z**.

We know the modes *a priory*, only the right constants need to be found[10].

[7] R. J. Garbacz, "A generalized expansion for radiated and scattered fields", PhD thesis, Department of Electrical Engineering, Ohio State University, 1968

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[10] D. G. Dudley, *Mathematical foundations for electromagnetic theory*. IEEE Press, 1994

Derived Analytical Results – Eigenvalues λ_n

Use of

$$\lambda_n = \frac{\int\limits_{\Omega} \boldsymbol{J}_n^* \cdot \mathcal{X}\left(\boldsymbol{J}_n\right) \mathrm{d}\Omega}{\int\limits_{\Omega} \boldsymbol{J}_n^* \cdot \mathcal{R}\left(\boldsymbol{J}_n\right) \mathrm{d}\Omega} \tag{6}$$

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- characteristic eigenvalues

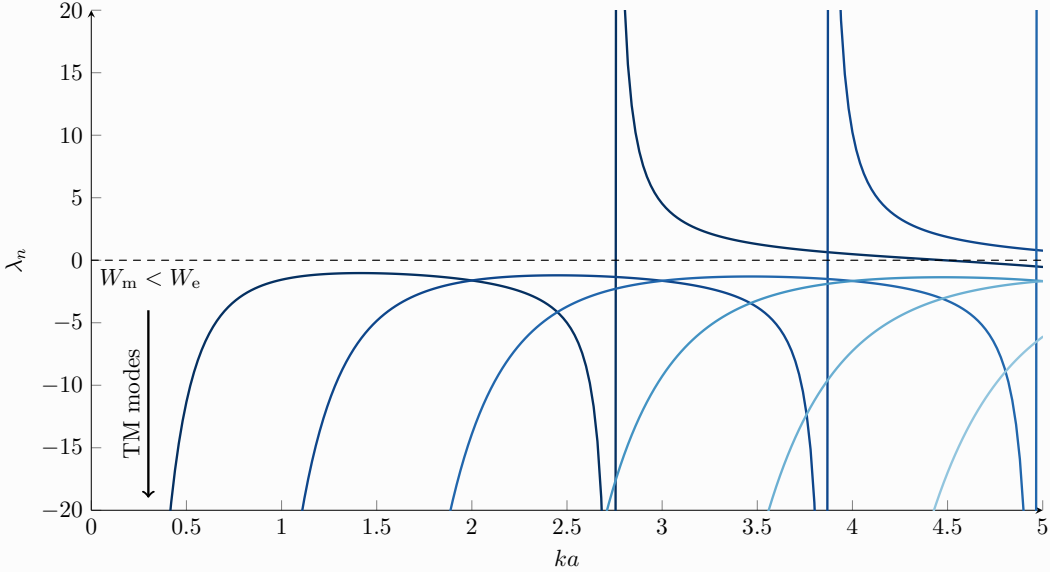
$$\lambda_n^{\mathrm{TE}} = -\frac{ka\,\mathrm{y}_n\left(ka\right)}{ka\,\mathrm{j}_n\left(ka\right)}, \tag{7}$$

$$\lambda_n^{\mathrm{TM}} = -\frac{(n+1)\,\mathrm{y}_n\left(ka\right) - ka\,\mathrm{y}_{n+1}\left(ka\right)}{(n+1)\,\mathrm{j}_n\left(ka\right) - ka\,\mathrm{j}_{n+1}\left(ka\right)}, \tag{8}$$

- characteristic eigenangles

$$\delta_n = 180\left(1 - \frac{1}{\pi}\arctan\left(\lambda_n\right)\right). \tag{9}$$

Eigenvalues λ_n



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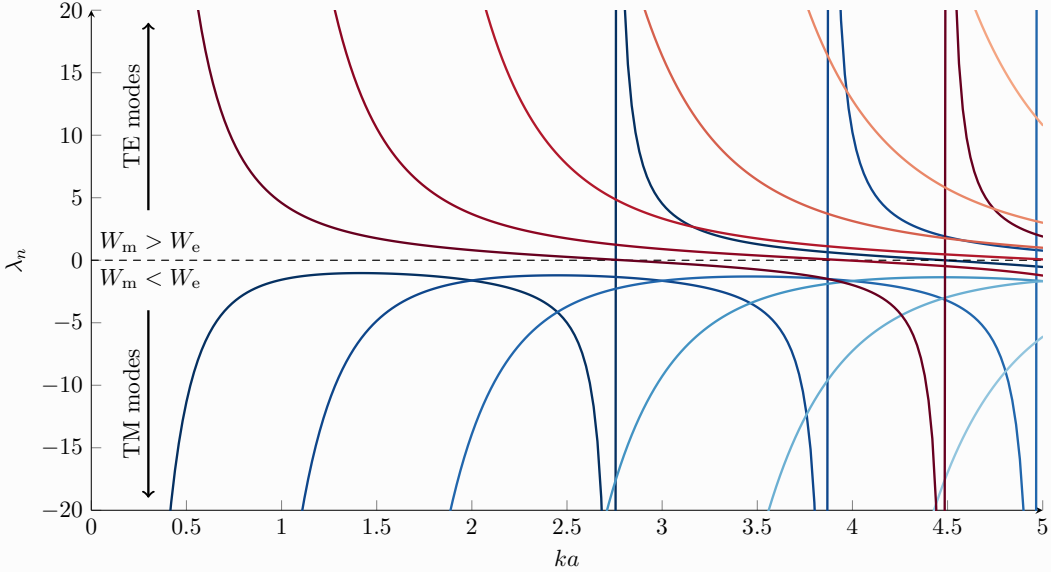

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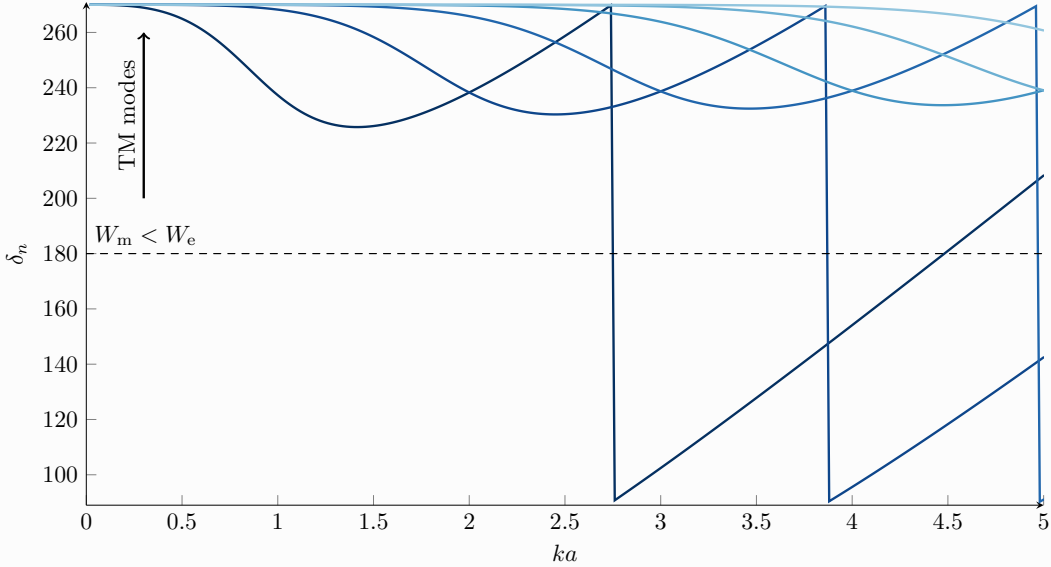

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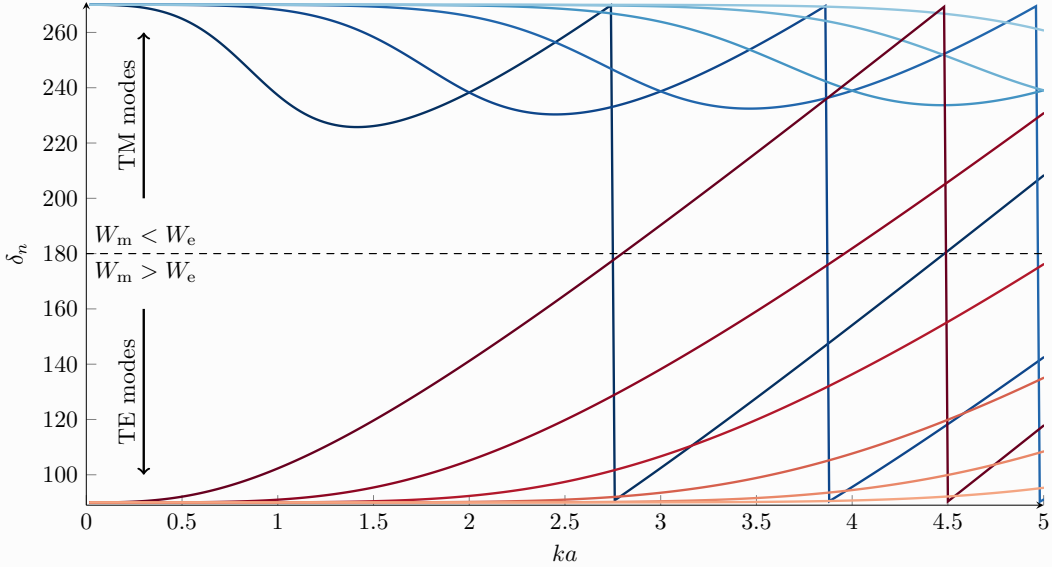


What Problems Can We Expect?

While simplest canonical body, spherical shell has plenty of potential issues, e.g.,

- degenerate eigenspace[11],
- conformity of spherical surface with commonly used basis functions,
- internal resonances[12],
- computationally demanding evaluation[13].

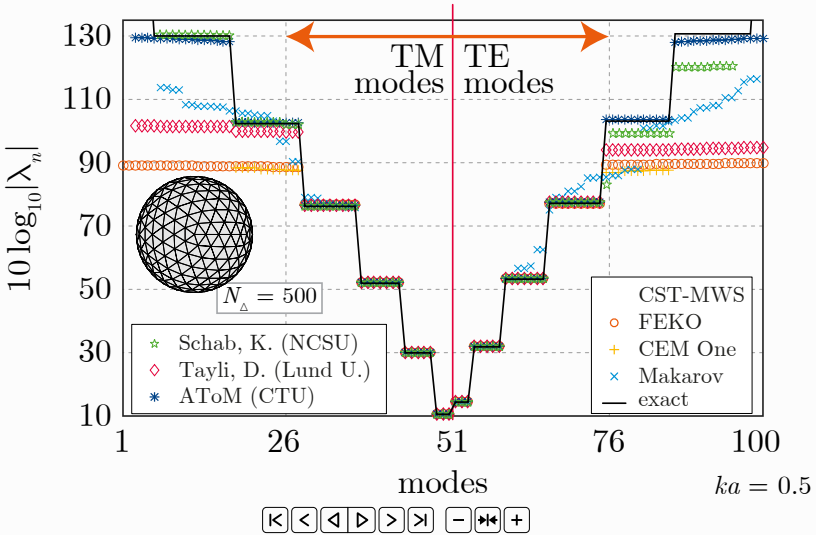
Spherical shell made of 194 triangles.

[11] K. R. Schab and J. T. Bernhard, "A group theory rule for predicting eigenvalue crossings in characteristic mode analyses", *IEEE Trans. Antennas Propag.*, no. PP, 2016, K. R. Schab, J. M. Outwater Jr., M. W. Young, *et al.*, "Eigenvalue crossing avoidance in characteristic modes", *IEEE Trans. Antennas Propag.*, vol. 64, no. 7, pp. 2617–2627, 2016. DOI: 10.1109/TAP.2016.2550098

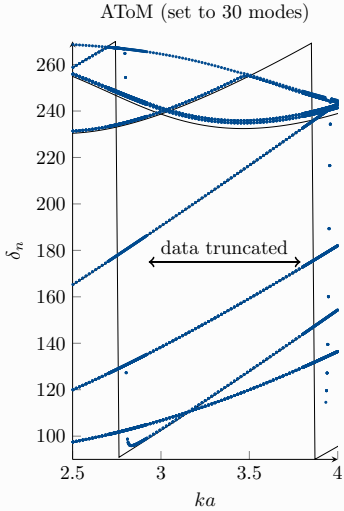
[12] T. K. Sarkar, E. Mokole, and M. Salazar-Palma, "An expose on internal resonance, external resonance and characteristic modes", *IEEE Trans. Antennas Propag.*, vol. PP, 2016. DOI: DOI10.1109/TAP.2016.2598281

[13] Having no junctions, spherical shell (and other closed objects) has highest possible ratio between number of basis functions (unknowns) and triangles (3/2).

Eigenvalues λ_n From Impedance Matrix



Characteristic Eigenangles Depending on ka



Data are processed differently, commercial packages are limited in number of modes, sphere of 500 triangles.

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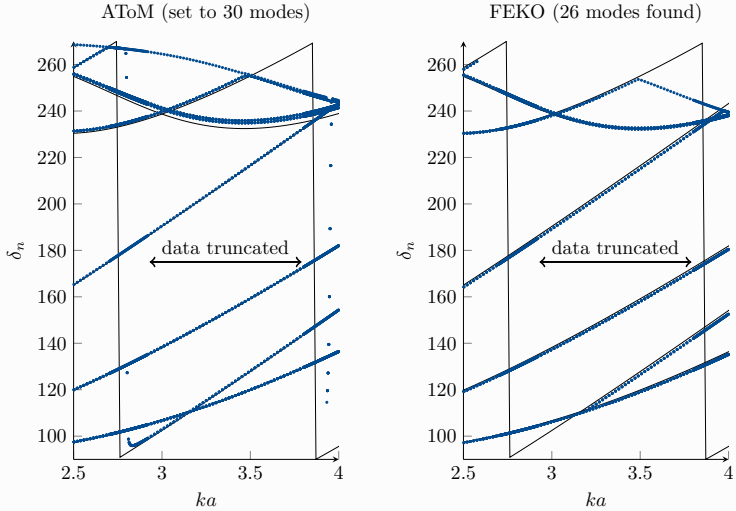

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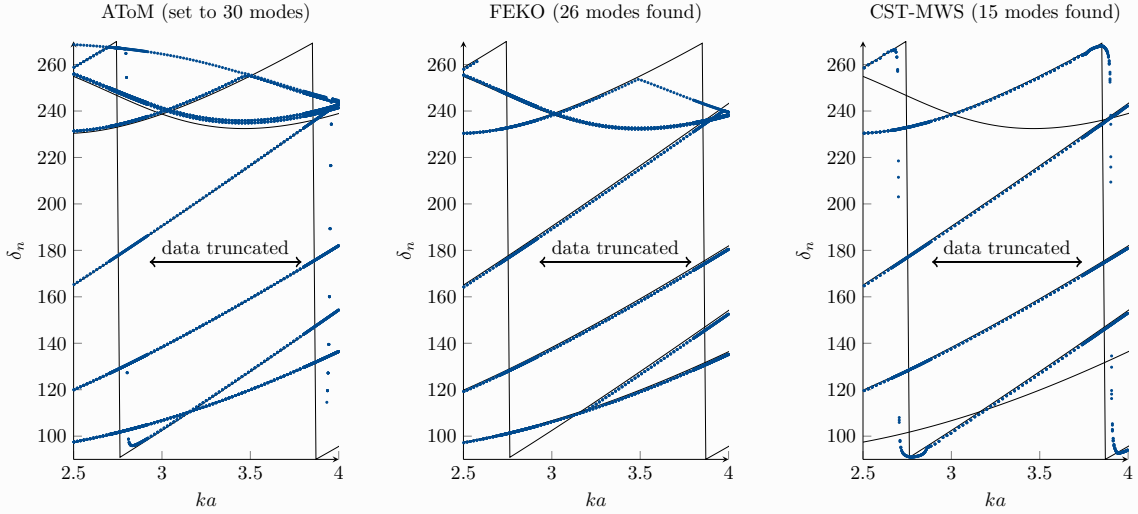

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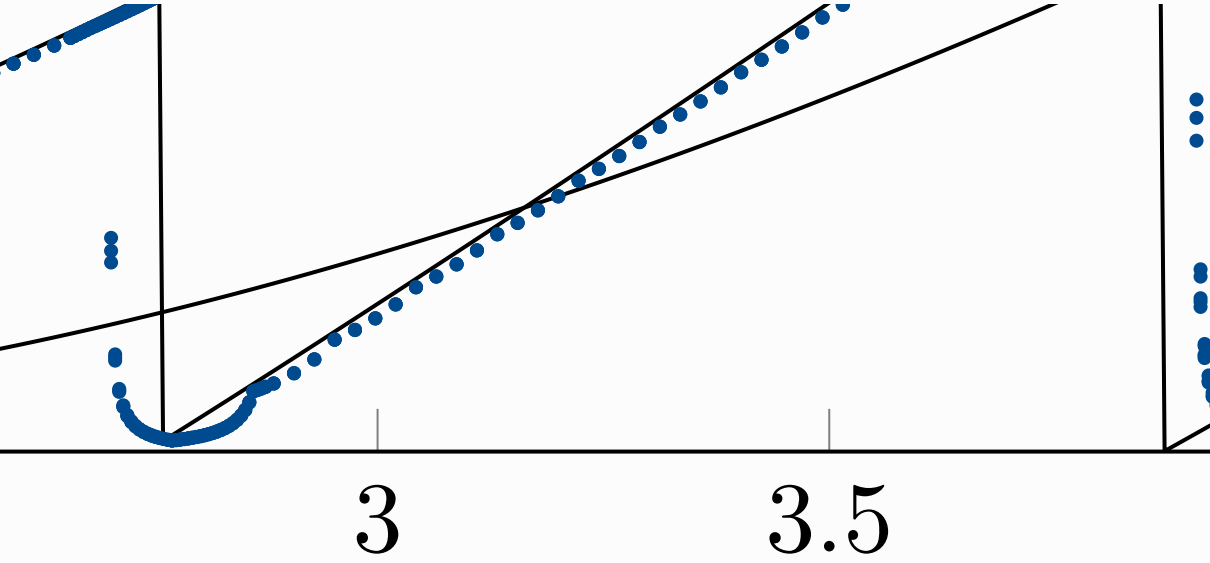

Observations

- CST-MWS uses CFIE[14] and
  - is extremely memory-consuming,
  - only 15 modes[15] have been obtained for $ka \in \{0.5, 5\}$
- CST-MWS uses interpolation
  - brief documentation
- no impedance matrix for CST-MWS
- FEKO uses EFIE
  - has surprising problems to track modes of sphere
  - only 26 modes[15] have been obtained for $ka \in \{0.5, 5\}$
- singularity treatment and used quadrature rule are crucial
  - done yet not presented

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[15] Consider the 3–5–7 degeneracy of first three TM, TE modes.

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Results can be used as *standard* for validating codes.

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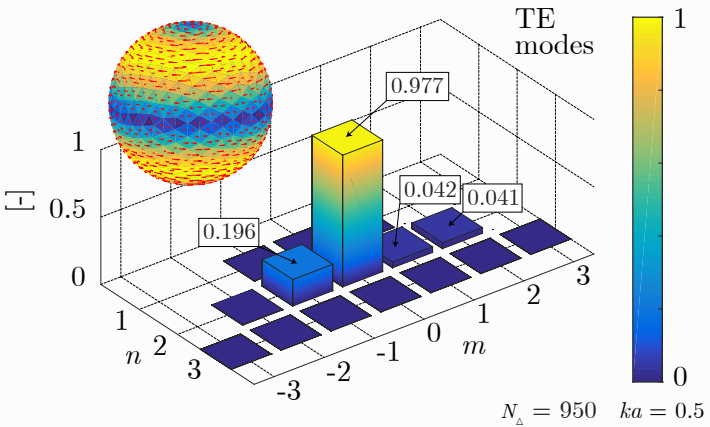
Study of Eigenmodes

Eigenmodes have to be studied carefully...

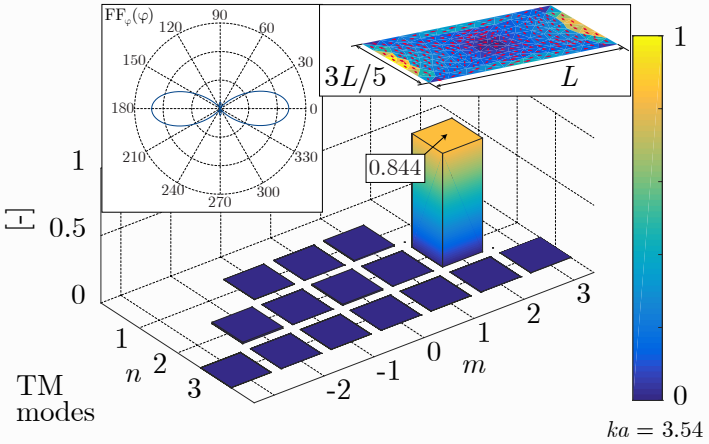


Projections of numerically calculated TE_{20} on set of analytical TE modes.

$$\xi = \int_{4\pi} \boldsymbol{F}_{\mathrm{CM}} \cdot \boldsymbol{F}_{nm}^{\mathrm{e/o}} \, \mathrm{d}S$$

- massive postprocessing needed (rotation of numerical or analytical currents)
- dependence on mesh density
- influence of density function
- low number of modes correlate well

By-product: Projection of Arbitrary Radiation Patterns



- shows how the small antenna behaves
- can be done in a clever way...

Projections of numerically calculated TE_{20} on set of analytical TE modes.

Further Work and Questions

Spherical shell makes it possible to test our codes against analytical data:

- meshing schemes, basis functions, singularities, integration rules,
- integral equations, materials (?),
- properties of CM decomposition, tracking routines
- hardware (CPU, RAM, HDD) requirements.

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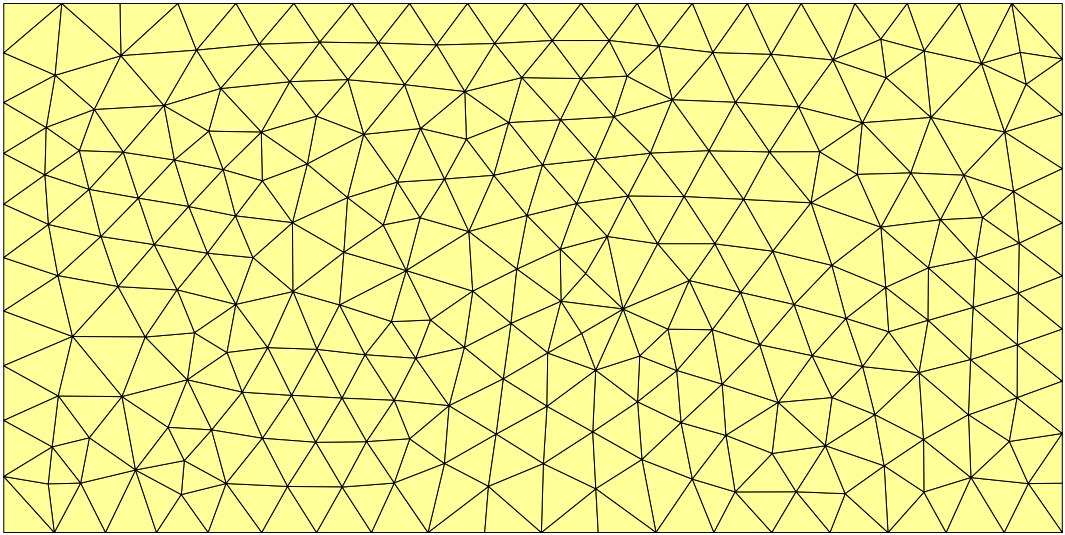
Ongoing work

- provide all formulas and results to the community, including gathered data,
- study the difference of between $\mathbf{S}$ (scattering matrix) and $\mathbf{Z}$ (impedance matrix) formulations – both are analytically solvable for spherical shell,
- study preconditioning $\mathbf{X}\mathbf{I}_n = \zeta_n \left(\mathbf{R} + \mathbf{\Sigma}\right) \mathbf{I}_n$ in the vicinity or resonances,
- think about solution to conical system,
- dream about (at least asymptotic solution) for spheroidal system.

Optimal Currents For Minimal Quality Factor Q

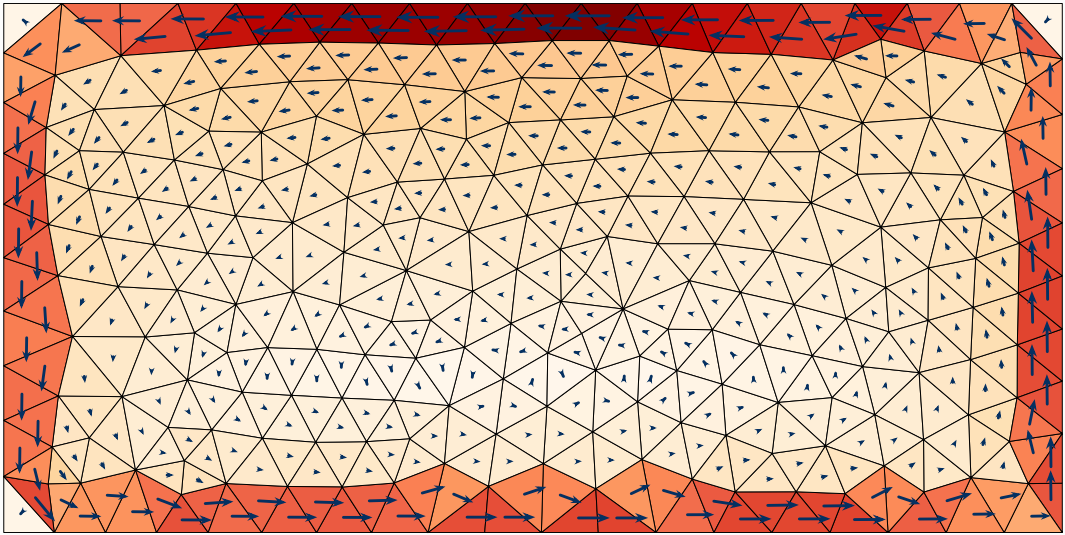
Take the pen and try to draw a current possessing minimal quality factor Q...



PEC plate $L \times L/2$, PEC, $ka = 0.5$.

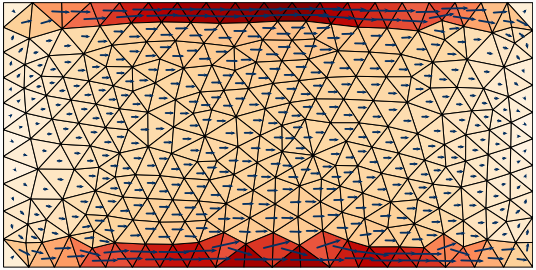
Optimal Currents For Minimal Quality Factor Q

. . . here is the correct answer.

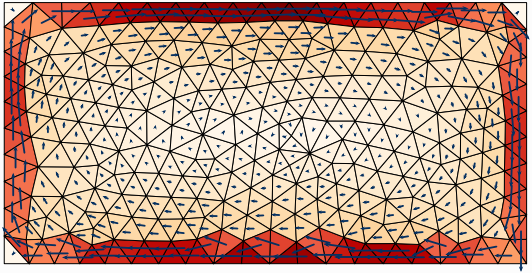


Optimal current with respect to minimum quality factor Q.

Modal Composition of Optimal Currents



Dominant (dipole-like) characteristic mode, $\alpha_1 = 1$.



First inductive (loop-like) mode, $\alpha_2 = 0.4553$.

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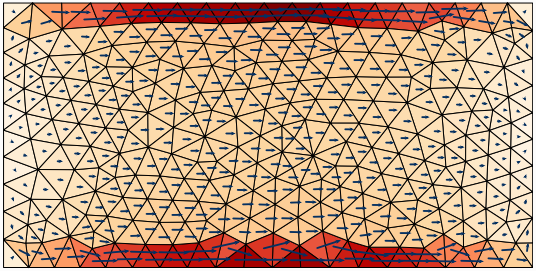

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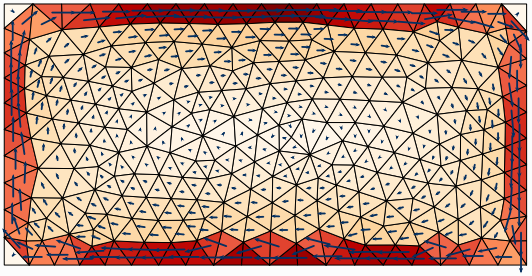


First inductive (loop-like) mode, $\alpha_2 = 0.4553$.

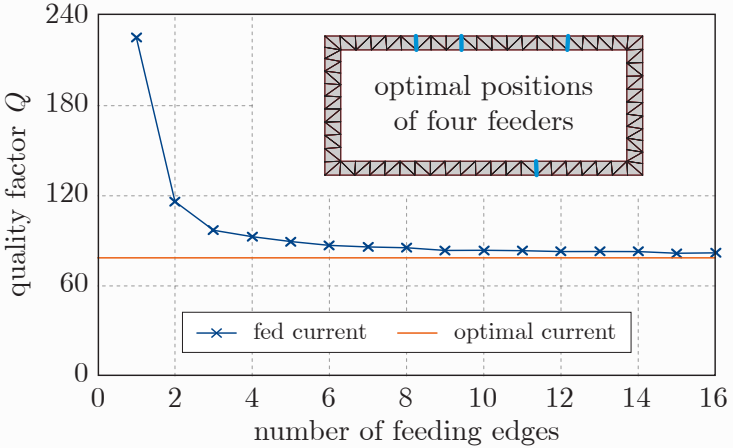
- CMs are quite convenient to define and interpret optimal currents[16], e.g., for $ka < 1$ we have $Q_{\min} = \mathcal{F}\left(\boldsymbol{J}_1 + \alpha_2 \boldsymbol{J}_2\right)$
- optimal currents yields the true bounds $\rightarrow$ they cannot be excited

[16] M. Capek and L. Jelinek, "Optimal composition of modal currents for minimal quality factor Q", , *IEEE Trans. Antennas Propag. (Early Access)*, vol. PP, 2016. DOI: 10.1109/TAP.2016.2617779. [Online]. Available: arXiv:1602.04808

How to Excite the Optimal Currents

- what about excitation of these currents?

How to Excite the Optimal Currents



Dependence of $Q_{\min}$ on number of (optimally placed) feeders

- what about excitation of these currents?
  - how many feeders do we need?
  - where they should be placed?
  - should the structure be changed?

Alternative Bases (Matrix Representation)

- CMs are excellent for scattering problems (MIMO)
- discrete excitation of a given current needs further development:

$$\mathbf{I} = \sum_n \frac{\mathbf{I}_n^{\mathrm{H}} \boldsymbol{V}}{1 + \mathrm{j}\lambda_n} \frac{\mathbf{I}_n}{\mathbf{I}_n^{\mathrm{H}} \mathbf{R} \mathbf{I}_n} \tag{10}$$

- some weak points from algebraic point of view (e.g., $\mathbf{R} \not\succ 0$)
- CMs form nice basis, however, we have other bases[17] as well, e.g.,

$$\omega \mathbf{W} \mathbf{I}_n = \frac{\kappa_n}{2} \mathbf{R} \mathbf{I}_n \tag{11}$$

$$\mathbf{R} \mathbf{I}_n = \kappa_n \left(\mathbf{R} + \boldsymbol{\Sigma} \right) \mathbf{I}_n \tag{12}$$

[17] L. Jelinek and M. Capek, "Optimal currents on arbitrarily shaped surfaces", *IEEE Trans. Antennas Propag. (Early Access)*, vol. PP, 2016, eprint arXiv:1602.05520

Questions?

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